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Prediction of the effective Young's modulus of a particulate composite containing fractured particles

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A B S T R A C T

This paper considers the problem of prediction of the effective Young's modulus of a particulate composite material containing fractured particles. It treats the general case in which some particles are fractured while others remain intact. The reinforcing particles are assumed to be spherical. The Mori–Tanaka model is extended to formulate the method of solution. The resulting auxiliary problem of a single fractured particle in an infinite matrix subjected to a remote stress equal to the average matrix stress, for which Eshelby's solution does not exist, is solved by the finite element method. The predictions are compared with the exact results of Young's modulus for particulate composites with body-centered cubic packing arrangement and experimental results of Young's modulus for particulate composites containing fractured particles.

1. Introduction

Particle fracture is one of the fundamental damage mechanisms of particle-reinforced composite materials. Stiffness reduction inevitably results from the presence of fractured particles. It is of interest to predict the stiffness change of particulate composites as a function of the extent of particle fracture, particularly from the viewpoint of damage-tolerant design of composite structures.

Previous studies on particulate composites with particle fracture include those by Sun et al. [1], Liu et al. [2], Kouzeli et al. [3], Segurado and LLorca [4], Gonzalez and LLorca [5], and Shen et al. [6]. Highly approximate solution of the effective Young's modulus of a composite containing fractured particles was given by Mochida et al. [7], in which the fractured particles are replaced by penny-shaped cracks.

Stiffness prediction is relatively simple for composite materials of periodic particle packing arrangement with all the particles identically fractured [6]. This permits the use of a repeating unit cell with periodic boundary conditions, and composite stiffness properties can be readily determined. However, damage is a progressive and cumulative process, and the case in which all the reinforcement particles are fractured represents only the final stage of the damage process. It is therefore important to determine the evolutionary change in stiffness properties with progressing particle fracture.

In this paper we consider the problem of predicting the effective Young's modulus of a particulate composite containing fractured particles. The paper treats the general case in which some particles are fractured while others remain intact. The Mori–Tanaka method is extended to determine the effective Young's modulus of the composite weakened by the fractured particles. This requires the solution of the average stress and strain of a single fractured particle in an infinite matrix under a remote stress field. The finite element method is used to obtain the solution. Note that the problem of determining the effective Young's modulus of a particulate composite containing fractured particles falls within the general subject of stiffness properties of cracked solid materials [8].

2. Formulation

Consider a particle-reinforced composite material containing fractured particles. The reinforcing particles are assumed to be spherical. The particles and the matrix are homogeneous, isotropic and linearly elastic, with perfect particle–matrix interface, and particle fracture is characterized by penny-shaped cracks oriented in the same direction and dividing each particle into equal halves. The composite effective Young's modulus in the direction normal to the particle crack faces is to be determined. We consider the general case in which some particles are fractured while others remain intact. Fig. 1 illustrates a particulate composite with both intact and fractured particles.

Define a Cartesian coordinate system (x_1, x_2, x_3), with the x_1 direction normal to the particle crack faces. When a

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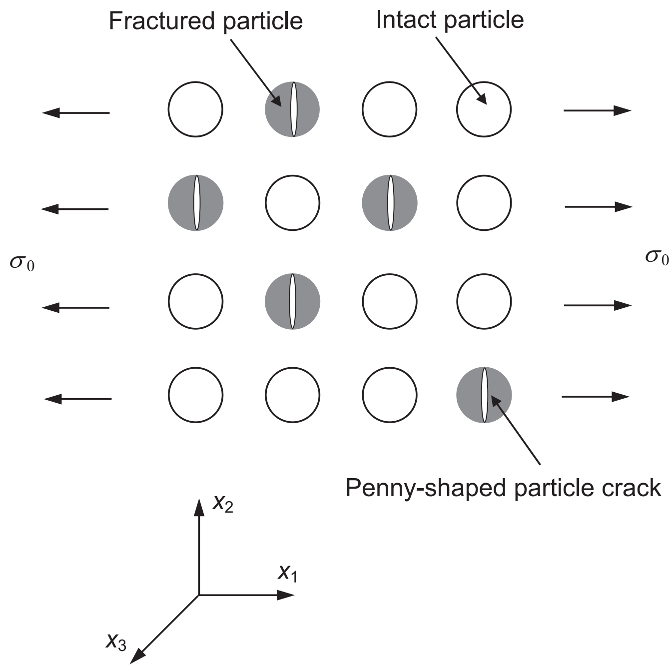

Fig. 1. A particulate composite material containing fractured particles under loading.

representative volume element of the composite is subjected to tractions $T_i=\sigma_{ij}^0 n_j$ consistent with a uniform stress field σ_{ij}^0 on its boundary surface with the outward unit normal n_i (i, j=1, 2, 3), the components of the composite volume average stress $\overline{\sigma}_{ij}$ are equal to the respective components of the applied uniform stress σ_{ij}^0 and can be expressed as

$$\overline{\sigma}_{ij}=\sigma_{ij}^0=(1-n_{fp})c_p\overline{\sigma}_{ij}^p+n_{fp}c_p\overline{\sigma}_{ij}^{fp}+(1-c_p)\overline{\sigma}_{ij}^m \quad i,j=1,2,3 \tag{1}$$

where $\overline{\sigma}_{ij}^p$ are the components of the average stress over the volume of the intact particles, $\overline{\sigma}_{ij}^{fp}$ are the components of the average stress over the volume of the fractured particles, and $\overline{\sigma}_{ij}^m$ are the components of the volume average matrix stress. In (1) c_p is the particle volume fraction, and n_{fp} is the fraction of fractured particles equal to the ratio of the volume of the fractured particles to the volume of all the particles. The fraction of fractured particles has values between 0 and 1, with the value 0 corresponding to all the particles being intact and the value 1 corresponding to all the particles being fractured.

The components of the volume average strain of the composite are given by

$$\overline{\varepsilon}_{ij}=(1-n_{fp})c_p\overline{\varepsilon}_{ij}^p+n_{fp}c_p\overline{\varepsilon}_{ij}^{fp}+(1-c_p)\overline{\varepsilon}_{ij}^m+\Gamma_{ij} \quad i,j=1,2,3 \tag{2}$$

where $\overline{\varepsilon}_{ij}^p$ are the components of the phase average strain over the volume of the intact particles, $\overline{\varepsilon}_{ij}^{fp}$ are the components of the phase average strain over the volume of the fractured particles, $\overline{\varepsilon}_{ij}^m$ are the components of the volume average strain of the matrix, and Γ_{ij} are the components of the part of the composite average strain attributed to particle cracks, which are given by

$$\Gamma_{ij}=\frac{1}{2V}\int_{S_{fp}}([u_i]n_j+[u_j]n_i)dS \tag{3}$$

In (3) V is the volume of the composite representative volume element, S_{fp} denotes the total area of the particle cracks, $[u_i]$ are the components of the displacement jump across the crack faces, and n_i are the components of the unit normal to the crack faces.

Since the terms Γ_{ij} (i, j=1, 2, 3) represent the part of the composite average strain due to particle cracking, it is more appropriate to express these terms using the volume of the fractured particles V_{fp} as follows.

$$\Gamma_{ij}=c_p n_{fp}\gamma_{ij} \tag{4}$$

with

$$\gamma_{ij}=\frac{1}{2V_{fp}}\int_{S_{fp}}([u_i]n_j+[u_j]n_i)dS \tag{5}$$

We describe γ_{ij} as the average strain of the fractured particles due to particle cracking. By using the stress–strain relations for the particle and the matrix material the composite average strain can be expressed as

$$\overline{\varepsilon}_{ij}=(1-n_{fp})c_pS_{ijkl}^p\overline{\sigma}_{kl}^p+n_{fp}c_pS_{ijkl}^p\overline{\sigma}_{kl}^{fp}+(1-c_p)S_{ijkl}^m\overline{\sigma}_{kl}^m+n_{fp}c_p\gamma_{ij} \tag{6}$$

where S_{ijkl}^p and S_{ijkl}^m (i, j, k, l=1, 2, 3) are the components of the elastic compliance tensors of the particle and the matrix material, respectively. Note that a repeated index indicates summation.

The average stresses in both the intact and the fractured particles, and the average strain of the fractured particles due to particle cracking as expressed in terms of the surface integral of the displacement jump can be evaluated by applying the Mori–Tanaka method as interpreted by Benveniste [9]. The underlying assumptions of the Mori–Tanaka approach in the present context are: (1) the average stress of the intact particles can be estimated by embedding a single intact particle in an infinite matrix subjected to a remote stress equal to the average matrix stress; (2) the average stress of the fractured particles and the average strain of the fractured particles due to particle cracking can be estimated by embedding a single fractured particle in an infinite matrix subjected to a remote stress equal to the average matrix stress.

The average stresses of the intact and fractured particles can be obtained by means of stress concentration tensors:

$$\overline{\sigma}_{ij}^p=B_{ijkl}\overline{\sigma}_{kl}^m \tag{7}$$

$$\overline{\sigma}_{ij}^{fp}=B_{ijkl}^{fp}\overline{\sigma}_{kl}^m \tag{8}$$

where B_{ijkl} and B_{ijkl}^{fp} are the components of the stress concentration tensors of the intact and fractured particles, respectively. Similarly the average strain of the fractured particles due to particle cracking can be written in terms of a concentration tensor with components D_{ijkl} as follows.

$$\gamma_{ij}=D_{ijkl}\overline{\sigma}_{kl}^m \tag{9}$$

The stress concentration tensor B_{ijkl} of the intact particles can be obtained through Eshelby's tensor as given in the Appendix. No such Eshelby's tensor exists for the fractured particles. To obtain the concentration tensors B_{ijkl}^{fp} and D_{ijkl} the problem of a fractured particle in an infinite matrix subjected to a stress field equal to $\overline{\sigma}_{ij}^m$ at infinity must be solved.

For convenience the following notations are used.

$$\sigma_1=\sigma_{11},\quad \sigma_2=\sigma_{22},\quad \sigma_3=\sigma_{33},\quad \sigma_4=\sigma_{23},\quad \sigma_5=\sigma_{31},\quad \sigma_6=\sigma_{12} \tag{10}$$

$$\varepsilon_1=\varepsilon_{11},\quad \varepsilon_2=\varepsilon_{22},\quad \varepsilon_3=\varepsilon_{33},\quad \varepsilon_4=2\varepsilon_{23},\quad \varepsilon_5=2\varepsilon_{31},\quad \varepsilon_6=2\varepsilon_{12} \tag{11}$$

$$\gamma_1=\gamma_{11},\quad \gamma_2=\gamma_{22},\quad \gamma_3=\gamma_{33},\quad \gamma_4=2\gamma_{23},\quad \gamma_5=2\gamma_{31},\quad \gamma_6=2\gamma_{12} \tag{12}$$

The components of the compliance tensors S_{ijkl}^p and S_{ijkl}^m are then reduced to S_{pq}^p and S_{pq}^m, respectively. Likewise the components of the concentration tensors B_{ijkl}, B_{ijkl}^{fp} and D_{ijkl} are reduced to B_{pq}, B_{pq}^{fp} and D_{pq}, respectively. And (1), (6), (7), (8) and (9) can be