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An isogeometric finite element formulation for thermal buckling analysis of functionally graded plates

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ABSTRACT

We address in this paper an isogeometric finite element approach (IGA) in combination with the third-order deformation plate theory (TSDT) for thermal buckling analysis of functionally graded material (FGM) plates. TSDT accounts shear deformation effect without requiring any shear correction factors. The IGA utilizes non-uniform rational B-spline (NURBS) as basis functions, resulting in both exact geometric representation and high order approximations. It enables to achieve easily the smoothness with arbitrary continuous order. The present method hence fulfills the C^1-requirement of TSDT model. The material properties of FGM plates are assumed to vary according to power law distribution of the volume fraction of constituents. The temperature field through the plate thickness is described by a polynomial series. The influences of length to thickness ratio, aspect ratio, boundary conditions and material property on the temperature critical buckling are investigated. Numerical results of circular and rectangular plates are provided to validate the effectiveness of the proposed method.

1. Introduction

Functionally graded materials (FGMs) proposed by a group of scientists in Sendai-Japan [1,2] are often made of a ceramic at the top and a metal at the bottom as shown in Fig. 1. Ceramic with the low thermal conductivity can resist high thermal environment while metal strongly supports mechanical load. Creating from these advantageous features of two above materials, FGMs have been using widely in many engineering structures corresponding to extremely high temperature environment such as aerospace, aircrafts, high-speed vehicles, nuclear plants, etc.

Due to its wide application, many researches about functionally graded structures have been carried out. For instance, Noda and Tsuji [3,4] investigated the steady thermal stresses in a functionally graded plate. Zhang et al. [5] studied thermal stresses of FGM plates around a circular hole. Reddy et al. [6,7] studied the behavior of functionally graded microstructure-dependent beams. Thermal behavior of FGM plates with temperature-dependent properties was reported by Lee et al. [8]. Vel and Batra [9,10] provided an analytical solution for thermo-mechanical deformations of FGM plates. Reddy [11] presented the Navier's solutions of rectangular FGM plates that consider thermo mechanical coupling, time dependency and the von Karma-type geometric non-linearity, etc. Besides static and free vibration analysis, thermal buckling problem plays very important role in practical application. For example, in some thin-walled structures such as beams [12], plates [13–17] or shells [18,19], the temperature rise caused by heating or friction produces in-plane compressible forces which make the structures to be buckled before reaching to a yield stress. At the buckling state, the structure survives large deformation behavior and reduces load carrying capacity. This paper thus focuses on the study of the buckling phenomena of FGM plates under change of temperature.

A theoretical framework of FGM plates is commonly relied on popular plate theories such as the Classical Plate Theory (CPT), First Order Shear Deformation Theory (FSDT) and High Order Shear Deformation Theory (HSDT). The CPT ignoring the transverse shear deformation cannot produce accuracy results for thick plates [20,21]. The FSDT [22–24] takes into account the effects of shear deformation and can be applied for both thick and thin FGM plates. However, the accuracy of solutions will be strongly dependent on the shear correction factors in which their values are quite dispersed through each problem. Hence, the HSDT that include higher-order terms in the approximation of the displacement field have been developed, e.g. Reddy [25,26], Matsunaga [27] and Kant and Babu [28,29]. These models disregard shear correction factors and yield more accurate and stable solutions (e.g. inter-laminar stresses and displacements [26,30]) than the FSDT. It is worth mentioning that some HSDT models [26] requires C^1-continuity of generalized displacements leading to the second-order derivative of the stiffness formulation and hence they cause some obstacles

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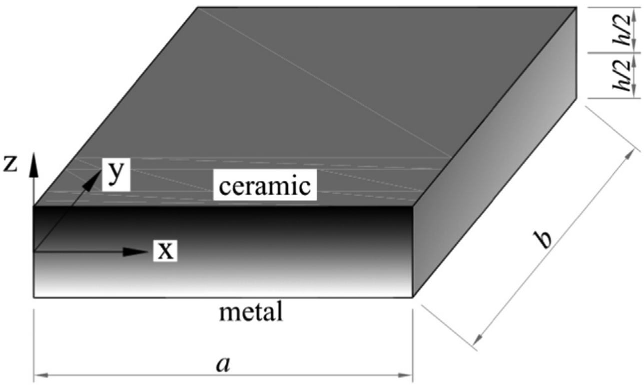

Fig. 1. The functionally graded plate model.

in standard finite formulations based on the C^0-continuity. In this paper, we will show that a C^1-HSDT formulation will be easily achieved using NURBS basis functions.

Isogeometric approach, a new computational method, has been proposed by Hughes et al. [33] to closely link the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). It means that the IGA uses the same basis functions for both the geometry description and the finite approximation. Being different from basis functions of the standard finite element method based on Lagrange polynomial, IGA utilizes more general basis functions such as Non-Uniform Rational B-splines (NURBS) that are common in CAD geometry. The exact geometry is therefore, preserved at the coarsest level of discretization and the re-meshing is then performed on this level without any communication with CAD geometry. Furthermore, NURBS provide a flexible way to make refinement, de-refinement, and degree elevation [34]. They can produce easily the smoothness of arbitrary continuity order in comparison with the traditional FEM. Hence, IGA naturally verifies the C^1-continuity of plates based on the HSDT assumption. IGA has been widely applied to a wide range of practical problems [35–43], and so on.

We address in this article a simple and efficient formulation relied on the framework of NURBS-based IGA for thermal buckling analysis of FGM plates under various thermal distributions. The IGA utilizes NURBS as basis functions, resulting in both exact geometric representation and high order approximations. It enables to achieve easily the smoothness with arbitrary continuous order. Hence, the present method naturally fulfills the C^1-requirement of HSDT model. We investigate critical temperature value versus various parameters such as length to thickness ratios, plate aspect ratios and properties of material. Numerical examples are provided to demonstrate the effectiveness of the present method.

The paper is outlined as follows. Next section describes problem model for FGM plates. In Section 3, we introduce a novel plate formulation based on IGA and TSDT. The numerical results and discussions are given in Section 4. Section 5 closes with some concluding remarks.

2. Problem model

2.1. Functionally graded material

Functionally graded materials (FGMs) are often formed by two or more different materials which properties change continuously along certain dimensions of the structure. Therefore, the properties of FGMs are derived from a function of position through

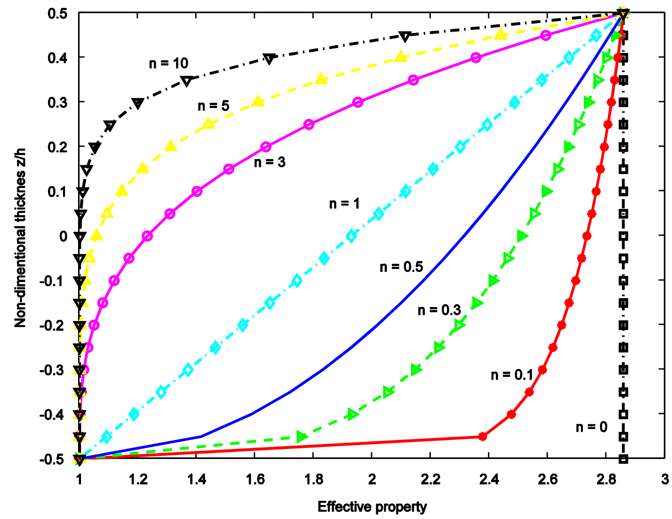

Fig. 2. The effective material property of Al/ZrO_2 FGM plate.

structural thickness based on the rule of mixture [11]:

$$P(z) = (P_c - P_m)V_c(z) + P_m$$
$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n \quad (1)$$

where $n \geq 0$ is the power law index; P_c, P_m denote the material properties of the ceramic and the metal at the top and bottom surfaces, respectively such as Young's modulus, thermal conductivity, Poisson's ratio, density, thermal expansion, etc.

Fig. 2 illustrates the distribution of Young's modulus through thickness of Al/ZrO_2 FGM plate via the power index n. It is observed that as $n=0$, the plate is fully ceramic with highest material property and as n increases the effective property is non-linearly distributed with decrease of magnitude through thickness.

In thermal analysis of the functionally graded plates, T_c and T_m denote the temperature at the top and bottom surfaces, respectively. From the one-dimensional steady state heat conduction equation, the temperature field through the plate thickness can be expressed by a polynomial series [17]:

$$T(z) = T_m + r(T_c - T_m)\sum_{i=0}^{\infty}\left(\frac{r^{ni}k_{mc}^i}{ni+1}\right) \Big/ \sum_{i=0}^{\infty}\left(\frac{k_{mc}^i}{ni+1}\right) \quad (2)$$

where

$$r = \frac{z}{h} + 0.5; \quad k_{mc} = \frac{k_m - k_c}{k_m} \quad (3)$$

with k is the thermal conductivity.

The temperature distribution through the thickness of the FGM plate via various values of the power law index n is illustrated in Fig. 3. It is evident that the temperature in the FGM plates distributes non-linearly and is always lower than that in the homogenous plates.

2.2. Governing equation for plate model

Based on the TSDT [25], the displacement field of an arbitrary point in the plate is defined as:

$$\begin{aligned} u(x,y,z) &= u_0 + z\beta_x + cz^3(\beta_x + w_{0,x}) \\ v(x,y,z) &= v_0 + z\beta_y + cz^3(\beta_y + w_{0,y}), \quad \left(\frac{-h}{2} \leq z \leq \frac{h}{2}\right) \\ w(x,y) &= w_0 \end{aligned} \quad (4)$$

where $c = -4/3h^2$ and the variables $\mathbf{u}_0 = \{u_0\ v_0\}^T$, w_0 and $\boldsymbol{\beta} = \{\beta_x\ \beta_y\}^T$ shown in Fig. 4 are the membrane displacements, the deflection of the mid-plane and the rotations in the x–z, y–z planes, respectively.