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Multiscale computation for transient heat conduction problem with radiation boundary condition in porous materials

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ABSTRACT

This paper reports a multiscale asymptotic analysis and computation for predicting heat transfer performance of periodic porous materials with radiation boundary condition. In these porous materials thermal radiation effect at micro-scale have an important impact on the macroscopic temperature field, which is our particular interest in this study. The multiscale asymptotic expansions for computing temperature field of the problem are constructed, and associated explicit convergence rates are obtained on some regularity hypothesis. Finally, the corresponding finite element algorithms based on the multiscale method are brought forward and some numerical results are given in details. The numerical tests indicate that the developed method is feasible and valid for predicting the heat transfer performance of periodic porous materials, and support the approximate convergence results proposed in this paper.

1. Introduction

Porous materials are widely applied in the engineering practice due to their various advantages, such as high heat resistance, low thermal conductivity, light weight, etc., which usually have multiple length scales [1–4]. Especially, with rapid development of space aircraft in recent years, porous materials designed as insulation for thermal protection system (TPS) have attracted tremendous attention and wide research interests from scientists and engineers [5–8]. As we know, coupled heat transfer mechanisms in porous materials used for engineering applications contain conduction, convection and radiation. The contribution of convection in total energy transfer was analyzed in Refs. [8–10]. It has been shown that convective heat transfer occurs by flow of fluids, and can be neglected at low pressures or in closed-cell porous materials [8,10]. Radiation is a way of heat transmission and plays a significant role in heat transfer at high temperature [8,10,11]. As a result, the dominant modes of heat transfer are conduction and radiation in porous materials. Generally, in reality, the coupled conduction and radiation problem in porous materials under various complex and extreme environments often makes the analysis and design of materials quite difficult. Also, the materials have periodic configurations and characteristic coefficients oscillate rapidly in small cells. Under such conditions, it is needed to develop new effective numerical and analytical technique for predicting the heat transfer performance of porous materials.

Solving the transient heat conduction problem with radiation boundary condition which arises in porous materials will be discussed. In this context, the direct accurate numerical computation of the solution becomes rather difficult because it would require a very fine mesh, and thus a prohibitive amount of computation time to effectively capture the local fluctuation behaviors of temperature field and their derivatives. We know that the homogenization method gives the overall behavior by incorporating the fluctuations due to the heterogeneities, which can not only save the computational resources but also ensure the calculation accuracy. Based on the mathematical homogenization method [12,13], various multiscale approaches for periodic problems have been proposed, refer to Refs. [14–19]. However, they only considered the first-order asymptotic expansions. In some cases, the homogenized solution and first-order solution are not enough to describe the local fluctuation in some physical fields, hence it is necessary to find higher-order multiscale asymptotic expansion for the solution. Recently, the higher-order multiscale asymptotic expansion developed to predict the physical and mechanical properties of composite materials has been discussed by Cui et al. [20–22]. By high-order correctors, the microscopic fluctuation of physical and mechanical behaviors inside the materials can be captured more accurately.

Up to now, some homogenization and multiscale methods were introduced for the coupled heat transfer problems in porous

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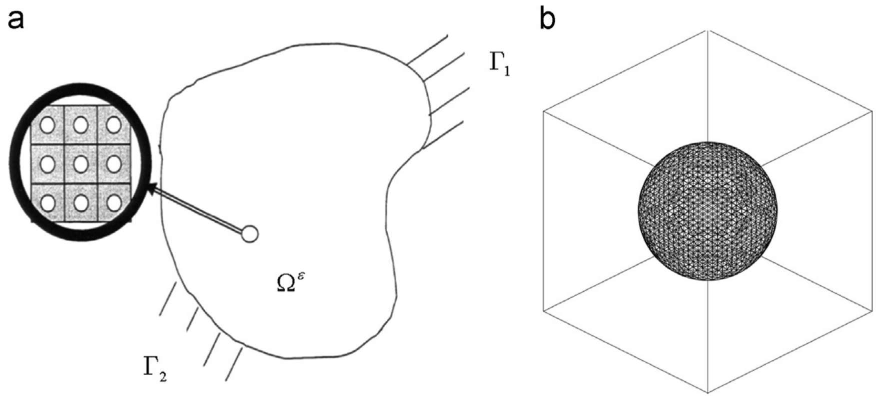

Fig. 1. Periodic distribution of porous materials: (a) domain Ω^ε and (b) unit cell Y^*.

materials. Amosov [23,24] investigated the nonstationary radiative–conductive heat transfer problem with rapidly oscillating coefficients, and obtained the error estimates. Liu and Zhang [25] predicted the effective macroscopic properties of heat conduction–radiation problem. Bakhvalov [26] obtained the formal expansions for the solution of those problems. On this basis, Allaire and El Ganaoui [27] discussed the heat conduction model with ε^{-1}-order radiation boundary conditions by two-scale asymptotic expansion, and justified the convergence. Meanwhile, Ma and Cui [28] proposed a higher-order asymptotic expansion method to solve the coupled problem, and derived the convergence results with an explicit rate $\varepsilon^{1/2}$. It should be noticed that this non-classical model cannot be used for the materials with low porosity, because it over-estimates the radiation behavior on the interior surface of cavities. Cui et al. [29,30] investigated the coupled conduction and radiation problem with small parameter ε, and obtained high-order expansions of the solution for the problem. However, to the best of our knowledge, an explicit convergence results and numerical algorithms for the heat conduction with more real radiation boundary conditions of parabolic equations were not found. So, this paper will lay a strong emphasis on the determination of the multiscale error analysis and numerical algorithms for the approximate solutions.

In the current work, we will mainly discuss the transient heat conduction problem of periodic porous materials with interior and exterior surface radiation. The interior radiation boundary condition was investigated by Liu and Zhang [25], Bakhvalov [26] and Cui et al. [29,30], and it is a classical model in physics. This paper is to establish a new high-order multiscale method to give a better approximation. We introduce correction terms into the first-order asymptotic expansion of the temperature field, define a family of cell functions, and then obtain the approximate error estimates under some regularity hypothesis. It should be pointed out that the error estimates in H^1-norm is still $O(\varepsilon^{1/2})$ due to its boundary error. Finally, some numerical results are given, which support the theoretical results of this paper and show the advantages and validity of the method developed.

The remainder of this paper is outlined as follows. Section 2 is devoted to the formulations of the multiscale asymptotic expansion for the coupled heat transfer problem. In Section 3 the error estimates on the approximate solution is analyzed. Finally, the multiscale algorithms and the numerical results for the heat transfer problem are shown, which support strongly our method.

Throughout the paper the Einstein summation convention on repeated indices is adopted. C denotes a positive constant independent of ε.

2. The multiscale asymptotic expansion

In this section, the multiscale asymptotic expansion formulations are constructed for solving the transient coupled heat transfer problem of periodic porous materials with radiation boundary condition.

Following Oleinik's notation (Ref. [13]), let $Y=\{y:0\le y_j\le 1, j=1,2,3\}$ and ω be an unbounded domain of R^3 which satisfies following conditions:

(B1) ω is a smooth unbounded domain of R^3 with a 1-periodic structure.
(B2) The cell of periodicity $Y^*=\omega\cap Y$ is a domain with a Lipschitz boundary, where Y^* is a reference periodicity cell, as shown in Fig. 1(b).
(B3) The set $Y\backslash\overline{\omega}$ and the intersection of $Y\backslash\overline{\omega}$ with the δ_0 neighborhood of ∂Yconsist of finite number of Lipschitz domains separated from each other and from the edges of the cube Y by a positive distance.
(B4) The cavities are convex.

The domain Ω^ε, as shown in Fig. 1(a), has the form: $\Omega^\varepsilon=\Omega\cap\varepsilon\omega$, where Ω is a bounded Lipschitz convex domain without cavities. Furthermore, suppose that the cavities surfaces are diffuse and gray, i.e., the emissivity e of the surfaces does not depend on the wavelength of the radiation, and assumes that the medium in the cavities is transparent. Here $\varepsilon>0$ is a small parameter representing the relative size of a periodic cell of porous materials, i.e., $0<\varepsilon=l_p/L\ll 1$, where l_p and L are respectively the sizes of a periodic cell and a whole domain Ω^ε are illustrated in Fig. 1(a) and (b).

The heat conduction equation with interior surface radiation with rapidly oscillatory coefficients is firstly studied by Bakhvalov [26], and later by Liu and Zhang [25] and Cui et al. [29,30], in which the radiation boundary condition in a closed cavity is essentially expressed as

$$-\nu_i k_{ij}^\varepsilon(x)\frac{\partial T_\varepsilon(x,t)}{\partial x_j}=e\sigma T_\varepsilon^4(x,t)-e\int_{\Gamma_{\varepsilon,m}^c}R_\varepsilon(z,t)F(x,z)dz,\quad \text{on } \Gamma_\varepsilon^c,$$

where $T_\varepsilon(x,t)$ denotes the temperature field, σ is the Stefan–Boltzmann constant, and $\mathbf{v}=(\nu_i)$, $i=1,2,3$ is the unit outward normal on Γ_ε^c. Γ_ε^c is the boundary that is composed of interior surfaces of cavity $\Gamma_{\varepsilon,m}^c$, such that $\Gamma_\varepsilon^c=\cup_{m=1}^{m(\varepsilon)}\Gamma_{\varepsilon,m}^c$, and $m(\varepsilon)$ the number of cavities contained in porous materials. $R_\varepsilon(x,t)$ is the intensity of emitted radiation defined by

$$R_\varepsilon(x,t)=e\sigma T_\varepsilon^4(x,t)+(1-e)\int_{\Gamma_{\varepsilon,m}^c}R_\varepsilon(z,t)F(x,z)dz,\qquad \forall x\in\Gamma_{\varepsilon,m}^c.$$

The classical radiation boundary condition is considered in this paper. To simplify the exposition, we assume that the emissivity is $e=1$. However, our analysis can be extended straightforwardly to the case $0<e<1$. In addition, the non linear radiation boundary condition that can be interpreted as a radiative heat exchange with