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A time increment control for return mapping algorithm applied to cyclic viscoplastic constitutive models

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ABSTRACT

This paper proposes two algorithms for better controlling the size of time increment in case of return-mapping integration scheme for elasto-viscoplastic constitutive models. Computation errors in terms of local stress–strain loops in Finite Element Analyses could indeed have a very strong impact on fatigue lifetime estimation protocol if time increment size are freely chosen by the FE solver. Proposed algorithm enables both to precisely describe the transition between elasticity and viscoplasticity and to avoid too important increase of time step during large viscoplasticity evolutions. The precision of the computed mechanical answer has been successfully tested for different kinds of multi-axial and anisothermal loading conditions on simple finite element and more complex meshed structures. Fatigue lifetime estimation errors have also been investigated for stabilized stress–strain cycle and common criteria and the proposed algorithm show very precise results with a limited computation time increase and without drastic limitation of the time increment size.

1. Introduction

For the past ten years, most of automotive manufacturers have adopted strategies mixing engine downsizing and mass reduction to answer to new environmental regulations and to decrease both polluting emission and fuel consumption. Engine parts, such as cylinder heads, exhaust manifolds and engine blocks, have therefore experienced higher operating temperatures induced by the increase of the combustion fluxes. They are therefore particularly sensitive to the superposition of a severe thermal loading and strong mechanical constraints that leads to thermal–mechanical fatigue cracks initiation.

Thermal–mechanical fatigue (TMF) design requires not only a good knowledge of the loading and boundary conditions [1,2] or of the thermal and mechanical environment of the studied structures but also advanced constitutive models which enable to precisely describe materials elasto-viscoplastic behaviors [3]. These models have to cope with various variable loading conditions and to describe complex anisothermal evolutions [4]. Consequently a higher level of sophistication and a greater number of parameters are often required to adequately represent the response induced by thermal–mechnanical loadings. In the framework of small perturbations and for metallic materials, lots of efforts have therefore been made to correctly describe the behavior of classic automotive alloys such as aluminiums, cast-irons or stainless steels with models taking into account ageing effects, stresses recovery or complex viscous behavior [5–9]. Moreover, the development of representative fatigue criterion remains an important research field as it constitutes the final step of a performant design process. Recent advances have been made by taking into account mean-stress effects [10,11] or by exploring the statistical influence of the microstructure [12].

The use of all of these models is standardly based on a Finite Element description of thermal–mechanical problems. Complex constitutive models must most of the time be integrated into commercial computation code as Abaqus in order to facilitate fatigue design operations within industrial engineering offices. The related evolution problem has received much attention mostly between the late seventies and the eighties, resulting in significant advances in the integration schemes [13–18]. For practical industrial applications and in the case of viscoplasticity, return mapping algorithms represent a very common scheme to integrate the rate constitutive equations [13, 19–23]. In this process, associated with a Newton iterative procedure, an elastic predictor is first estimated before being corrected onto a suitably updated yield surface. In order to obtain a quadratic rate of convergence, a consistent tangent operator [14,16,24] must also be used. Even if this class of algorithms has proven its numerical performance [25,2], it appears to be sensitive to the size of the chosen time increment. Others proposals such as substepping algorithms [26] have proved

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to be more accurate for large time increments but no relevant solution is proposed in the literature to efficiently control the increment size within the framework of radial return algorithm.

This paper proposes two algorithms for controlling the time step and thus better control both the calculated stress–strain loop for elasto-viscoplastic constitutive models [27] in the case of anisothermal thermomechanical loads and with a good compromise between CPU time and reliability of results. Drifts in terms of mechanical behavior are evaluated for different types of multiaxial loadings first on an volumic finite element and then a more complex structure. Constitutive models are developed for design of structures against thermomechanical fatigue and errors in terms of dissipated energy density, a commonly used indicator to assess the damage associated with this type of fatigue, are also estimated. The contribution of the proposed algorithms in terms of accuracy and computation time savings is finally considered in comparison with an excessive limitation of the time increment.

2. Constitutive model and non-linear analysis

In order to describe the behavior of metallic body-centered cubic alloy over a wide range of strain, strain rate and temperature, a constitutive model was proposed by Szmytka et al. [27]. This model is here taken as reference for the proposed algorithms which can easily be extended to other constitutive laws. Basic assumptions are small strains, isotropic elastic-viscoplastic behavior and unified elastic-viscoplasticity thermodynamic framework. Consequently, the total strain tensor is divided additively into elastic, inelastic and and thermal parts,

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^{e} + \underline{\underline{\varepsilon}}^{ine} + \underline{\underline{\varepsilon}}^{th} \tag{1}$$

The thermal strain is $\underline{\underline{\varepsilon}}^{th} = \alpha_T (T - T_{ref}) \underline{\underline{1}}$, while the elastic strain is given by Hooke's law,

$$\underline{\underline{\varepsilon}}^{e} = \mathbb{C}^{-1} : \underline{\underline{\sigma}} \tag{2}$$

where T is the temperature, T_{ref} is the reference temperature, $\underline{\underline{\sigma}}$ is the stress tensor. α_T is the thermal expansion coefficient and $\mathbb{C}$ is the fourth-order elastic tensor. In our isotropic case, $\mathbb{C} = \lambda \underline{\underline{1}} \otimes \underline{\underline{1}} + 2\mu \mathbb{I}$, with λ and μ are the Lamé constants dependent of the temperature. $\underline{\underline{1}}$ and I are respectively the second and the fourth order unit tensors. A viscoplastic equation with back stress is used to describe the evolution of the viscoplastic strain tensor,

$$\underline{\underline{\dot{\varepsilon}}}^{vp} = \frac{3}{2} \dot{p} \frac{\underline{\underline{s}} - \underline{\underline{X}}}{J_2(\underline{\underline{s}} - \underline{\underline{X}})} \tag{3}$$

where $\dot{(\,)}$ denotes the differentiation with respect to time, $\underline{\underline{s}}$ is the deviator of the stress tensor $\underline{\underline{\sigma}}$ and $\underline{\underline{X}}$ is the back stress tensor. $J_2(\underline{\underline{s}} - \underline{\underline{X}})$ is the second invariant of the tensor $\underline{\underline{s}} - \underline{\underline{X}}$ and is defined by using $(:)$ as the inner product between second rank tensors,

$$J_2(\underline{\underline{s}} - \underline{\underline{X}}) = \sqrt{\frac{3}{2} (\underline{\underline{s}} - \underline{\underline{X}}) : (\underline{\underline{s}} - \underline{\underline{X}})} \tag{4}$$

while $\dot{p}$ is the accumulated viscoplastic strain or viscoplastic multiplier. Its evolution, represented by a flow rule, follows the rule proposed by [27]

$$\dot{p} = \dot{\varepsilon}_0 \left(1 + \left(\frac{f(\underline{\underline{s}} - \underline{\underline{X}}, R)}{H} \right)^2 \right) \sinh \left[\frac{f(\underline{\underline{s}} - \underline{\underline{X}}, R)}{K} \right]_+ \left(1 - \exp \left[- \left[\frac{f(\underline{\underline{s}} - \underline{\underline{X}}, R)}{\beta R} \right]_+ \right]^m \right) \tag{5}$$

where $\dot{\varepsilon}_0$, H, K, m, β are temperature dependent parameters; $[.]_+$ indicates the positive part. The yield function f is defined as

$$f(\underline{\underline{s}} - \underline{\underline{X}}, R) = J_2(\underline{\underline{s}} - \underline{\underline{X}}) - R \tag{6}$$

R is the isotropic hardening term, and is considered here as following a conventional non-linear rule [28]:

$$R = R_0 + Q(1 - \exp(-bp)) \tag{7}$$

where R_0, Q and b are the three temperature-dependant parameters. Finally, for the evolution of the kinematic hardening term $\underline{\underline{X}}$, the conventional non-linear Armstrong and Frederick's law is adopted [28],

$$\underline{\underline{X}} = \frac{2C}{3} \underline{\underline{\alpha}} = C \left(\frac{2}{3} \underline{\underline{\dot{\varepsilon}}}^{ine} - \frac{D}{C} \underline{\underline{X}} \dot{p} \right) \tag{8}$$

where C and D are two material parameters. The constitutive model was shown in [27] to accurately reproduce the behavior of spheroidal graphite cast iron.

The solution of non-linear evolution problems such as cyclic viscoplasticity is performed, during implicit finite element analysis, over discrete sequence of time steps [25]. For each time step, predictor-corrector algorithms are used. In fact, the return mapping algorithm uses an operator split approach and an iterative Newton procedure consisting in solutions of global linear problem (elastic prediction) followed by local integrations of the constitutive model (plastic correction). A fully implicit integration scheme (backward Euler) associated with a radial return is adopted by Szmytka et al. [2] to successfully perform the numerical integration of the previous law in a material behavior user subroutine UMAT which is written and used in the commercial software Abaqus.

The main assumption of this scheme is that the viscoplastic strains evolution over the increment can be calculated from the stress state at the end of the increment. This is however theoretically incorrect as the viscoplastic response and in particular the viscoplastic flow direction is a function of the current stress state corrected by the kinematic hardening term. If the viscoplastic flow direction does not change over an increment, the return algorithm solutions will be accurate. However, although the time step is fixed sufficiently small to avoid viscoplastic direction variations, standard commercial finite element software tend to steadily increase it if convergence of the overall mechanical problem required only few iterations, introducing computations errors.

In fact, if one considers the evolution of the accumulated viscoplastic strain over a time step between t_n and t_{n+1}:

$$\Delta p_n = p_{n+1} - p_n = \int_{t_n}^{t_{n+1}} \dot{p}(u) du \tag{9}$$

the flow rule, as a continuous fonction of time, becomes

$$\Delta \underline{\underline{\varepsilon}}^{vp} = \underline{\underline{\varepsilon}}^{vp}_{n+1} - \underline{\underline{\varepsilon}}^{vp}_{n} = \int_{t_n}^{t_{n+1}} \underline{\underline{\dot{\varepsilon}}}^{vp}(u) du = \frac{3}{2} \int_{t_n}^{t_{n+1}} \dot{p} = (u) \frac{\underline{\underline{s}}(u) - \underline{\underline{X}}(u)}{J_2(\underline{\underline{s}}(u) - \underline{\underline{X}}(u))} du \tag{10}$$

$$= \frac{3}{2} \int_{t_n}^{t_{n+1}} \dot{p}(u) \underline{\underline{N}}(u) du \tag{11}$$

where $\underline{\underline{N}}(u)$is the tensor normal to the convex surface of visoplasticity. If $\underline{\underline{N}}(u)$ does not vary over $[t_n; t_{n+1}]$, the integration scheme is perfectly accurate and no error are introduced. However, if $\underline{\underline{N}}(u)$ varies during the time increment, there is a difference between the continuous definition of $\Delta \underline{\underline{\varepsilon}}^{ine}$ and its discrete value $\Delta \underline{\underline{\varepsilon}}^{ine} = \Delta p_n \frac{3}{2} \underline{\underline{N}}(t_{n+1})$, which leads to an integration error $\tilde{e}$:

$$\tilde{e} = \frac{3}{2} \int_{t_n}^{t_{n+1}} \dot{p}(u) \left[\underline{\underline{N}}(u) - \underline{\underline{N}}(t_{n+1}) \right] du \tag{12}$$

It is then necessary to observe and quantify the errors produced by this integration scheme for various loads and for different time increments as performed for example by [29,30] using the concept of iso-error maps. While the tested constitutive model is often used for thermal–mechanical fatigue design, errors