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Efficient hyper reduced-order model (HROM) for parametric studies of the 3D thermo-elasto-plastic calculation

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ABSTRACT

This paper focuses on a 3D thermo-elasto-plastic localized thermal source simulation and its parametric analysis with high CPU efficiency in the reduced-order model (ROM) framework. The hyper reduced-order model (HROM) is introduced and improved with two choices. Firstly, three reduced bases are constructed: one for the displacement increments, one for the plastic strain increments and one for the stress state. Equilibrium equation in plasticity relies on the knowledge of plastic strain rate, hence the plastic strain has to be included into the variable to be reduced, and the incremental form is adopted in the paper. It is shown that the introduction of an extra stress basis greatly improves the quality and the efficiency of the ROM. Secondly, the reduced state variables of plastic strain increments are determined in a reduced integration domain. Concerning the parametric analysis, the interpolation of the reduced bases is based on the Grassmann manifold, which permits to generate the new proper orthogonal decomposition bases for the modified parameters. In order to increase the convergence rate, the plastic strain interpolated from snapshots (the reference cases with full FEM calculations) is considered as the initial value of each time step for the modified problem of parametric studies. As a result, the plastic calculation is always done on the confined domain and only a few iterations are then required to reach static and plastic admissibility for each time step. The parametric studies on varying thermal load and yield stress show high versatility and efficiency of the HROM coupled with Grassmann manifold interpolation. A gain of CPU time of 25 is obtained for both cases with a level of accuracy smaller than 10%.

1. Introduction

In design and control of complex continuum mechanical or physical processes, standard simulation techniques using the finite element method (FEM) are not very efficient due to the large number of degrees of freedom (DOF) and time steps. Indeed, they can involve prohibitive global CPU. Hence, the reduced-order model (ROM) technique helps to save the computational cost using smaller number of DOF through capturing the significant transformations of the parameters of interest [1]. Among ROMs, the proper orthogonal decomposition (POD) reduction method has shown its efficiency for optimization problems in fluid mechanics [2], structural dynamics [3], material science [1], thermal science [4] and real time surgery simulation [5].

However, the classical ROM based on the Galerkin formulation by a POD basis is not efficient for medium size elasto-plastic problems. In such problems, the computational effort related to the local integration of the nonlinear constitutive laws, which is used to determine the state of a system involving internal variables (IVs), can represent more than 80% of the total computational effort [6]. The complexity of the local computations does not depend on the reduced order of the model, as the computational effort needed to estimate IVs is not related to the reduced approximation, and all the elements are included in the loops for obtaining the good estimations of IVs. This defeats the purpose of model reduction and limits the efficiency of the reduced POD models.

By selecting accurately a small set of elements from the detailed model and considering the related equilibrium conditions, it is possible to define a reduced integration domain (RID) for the mechanical model as proposed by Ryckelynck [6]. As a result, the estimation of the significant IVs is only limited within the RID, the remaining variables can be extrapolated to the whole domain by the reduced basis of the IVs, this technology is similar to the Gappy POD

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method [7]. This clever approach named hyper reduced-order model (HROM) enables the further computational reduction compared with the POD reduction model. However, the time saving is only limited to 75% in the elasto-plastic calculation. Later, Ryckelynck and Benziane [8] developed the A Priori Hyper-Reduction method (APHR) for nonlinear mechanical problems involving internal variables, and a multi-level formulation is introduced to focus on the capability of the method to perform efficient parallel computations, where the best computational saving of 97% is obtained.

Besides, a modified POD strategy of the empirical interpolation method (EIM) is suggested by Maday et al. [9], which reduces the evaluation of the nonlinear term in the reduced model to a level proportional to the number of POD-based reduced variables. Based on the EIM, the discrete empirical interpolation method (DEIM) is proposed by Chaturantabut and Sorensen [10]. The essential of EIM or DEIM is to replace the orthogonal projection in the Galerkin scheme with an interpolatory projection within the selected interpolation points for approximating the nonlinearities. Similar to the HROM, the high efficiency is obtained as it only evaluates the nonlinearities at a few interpolation points.

Concerning the elasto-plastic problem, Radermacher and Reese [11] proposed an adaptive method of sub-structure based on the selective POD, where the reduction is only applied in sub-domains with approximately elastic behavior. Then the POD basis is adapted with the selected sub-domain, while further research is needed on the efficiency. Moreover, the POD-based reduction method has been also extended to inelastic structures including cracks [12], where the global model reduction strategy is developed by mixing both the a posteriori and a priori approaches to simulate the crack propagation. Moreover, the damage initiation problems are also addressed by the POD hyper reduction model when strong topological changes are involved [13].

Recently, Relun et al. [14] have developed a reduction model based on the proper generalized decomposition (PGD) method for elastic-viscoplastic computational analysis, but similar computational time is found compared with FEM. In addition, the reference points method (RPM) is proposed by Ladeveze et al. [15] to decrease the computational complexity of algebraic operations in the framework of the PGD.

Generally speaking, the reduced methods of POD and PGD show potentials to decrease the computational efforts for the elasto-plastic problems, while the hyper reduced-order model (HROM) on thermo-elasto-plastic calculation has not been reported. This paper focuses on the HROM applied to highly localized 3D thermo-elasto-plastic calculation. The first aim of the present paper is to reduce the computational effort significantly by the HROM. Moreover, the precision of stress field is improved by using an additional stress basis in order to recover more efficiently balance equations such as the equilibrium equation. The second aim of the present paper is to perform parametric studies for the parameters of interest. The Grassmann manifold interpolation is then adopted for the adaptive POD basis corresponding to the modified parameter, and the known plastic strains of the the reference cases with full FEM calculations are considered as the initial value of the plastic calculation to accelerate the convergence rate for the parametric studies.

The 3D thermo-elasto-plastic calculation is presented in the context of hyper reduced-order model coupled with Grassmann manifold interpolation, and the structure of this paper is organized as follows: firstly, the thermo-elasto-plastic reference problem is presented. Secondly, the hyper reduced-order model and its improvement is introduced, the evaluation criteria of the POD basis number are developed. The high efficiency of the improved HROM is validated by comparing with the reference solution. Thirdly, the Grassmann manifold interpolation is presented for determining the POD basis corresponding to the modified value of the parameter of interest. Fourthly, the parametric studies on the variational thermal loading and yield stress are performed to validate the accuracy and the versatility of the proposed method.

2. Problem description

In this section, a structure (Ω) which obeys to usual thermo-elasto-plastic equations is employed. The thermo-elasto-plastic process subjected to a transient thermal load and usual boundary condition is typically assumed to weakly coupled. The temperature profile is assumed to be independent of stresses and strains. Therefore a heat transfer analysis is performed initially, and the results are imported for the stress analysis. The material constitutive model and the equilibrium equation are given in the below sub-section.

2.1. Material constitutive model

A linear isotropic hardening is considered for the thermo-elasto-plastic calculation. The strain rate tensor $\dot{\underline{\underline{\boldsymbol{\varepsilon}}}}$ is split into the elastic, plastic and thermal parts: $\dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{e}$, $\dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{p}$ and $\dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{th}$, respectively:

$$\dot{\underline{\underline{\boldsymbol{\varepsilon}}}} = \dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{e} + \dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{p} + \dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{th} \tag{1}$$

The equivalent plastic strain (PEEQ) p with the plastic multiplier is defined by

$$\dot{p} = \| \tfrac{2}{3} \dot{\underline{\underline{\boldsymbol{\varepsilon}}}}^{p} \| = \dot{\lambda} \tag{2}$$

The free energy is assumed to be of the following form:

$$\omega = \tfrac{1}{2} \left(\underline{\underline{\boldsymbol{\varepsilon}}} - \underline{\underline{\boldsymbol{\varepsilon}}}^{p} - \underline{\underline{\boldsymbol{\varepsilon}}}^{th} \right)^{T} : \underline{\underline{\underline{\underline{\mathbf{D}}}}} : (\underline{\underline{\boldsymbol{\varepsilon}}} - \underline{\underline{\boldsymbol{\varepsilon}}}^{p} - \underline{\underline{\boldsymbol{\varepsilon}}}^{th}) + \tfrac{1}{2} Hp^{2} \tag{3}$$

where $\underline{\underline{\underline{\underline{\mathbf{D}}}}}$ and H are Hooke's tensor and hardening modulus for linear isotropic hardening, respectively. The thermal strain can be defined by $\underline{\underline{\boldsymbol{\varepsilon}}}^{th} = \alpha \Delta T \underline{\underline{\mathbf{I}}}$, where α is the thermal expansion coefficient, and ΔT is the temperature increment.

The yield surface is defined by the function f:

$$f(\underline{\underline{\boldsymbol{\sigma}}}, p) = \sqrt{\underline{\underline{\boldsymbol{\sigma}}} : \underline{\underline{\underline{\underline{\mathbf{M}}}}} : \underline{\underline{\boldsymbol{\sigma}}}} - (\sigma_0 + Hp) \tag{4}$$

where σ_0 is the initial yield stress, and $\underline{\underline{\underline{\underline{\mathbf{M}}}}}$ is the stress operator tensor for von Mises calculation. It should be mentioned that a fixed-point type iteration strategy is suggested for the determination of the plastic multipliers λ [16]. Once λ is determined, the plastic strain $\underline{\underline{\boldsymbol{\varepsilon}}}^{p}$ can be obtained, then the stress can be expressed by

$$\begin{aligned} \underline{\underline{\boldsymbol{\varepsilon}}}^{e} &= \underline{\underline{\boldsymbol{\varepsilon}}} - \underline{\underline{\boldsymbol{\varepsilon}}}^{th} - \underline{\underline{\boldsymbol{\varepsilon}}}^{p} \\ \underline{\underline{\boldsymbol{\sigma}}} &= \underline{\underline{\underline{\underline{\mathbf{D}}}}} : \underline{\underline{\boldsymbol{\varepsilon}}}^{e} \end{aligned} \tag{5}$$

2.2. FE discretization and equilibrium equation

The discretized FE weak form for the thermo-elasto-plastic problem involves

$$\mathbf{f}^{int}(\mathbf{u}_{t_{n+1}}) = \mathbf{f}^{ext}(t_{n+1}) \tag{6}$$

where we introduce the state variable $\mathbf{X}_{(t_n)} = \{\mathbf{u}\ \boldsymbol{\varepsilon}^{p}\ \sigma\}_{n}^{T}$, which is known at time step t_n, while $\mathbf{X}_{(t_{n+1})} = \{\mathbf{u}\ \sigma\ \boldsymbol{\varepsilon}^{p}\}_{n+1}^{T}$ is not known.

In this case, the thermal and mechanical properties are weakly coupled. In a first stage, the temperature field $\mathbf{T}(t_{n+1})$ is calculated and the external force $\mathbf{f}^{ext}(t_{n+1})$ at time step t_{n+1} is given. Then the corresponding internal and external generalized node forces for a given degree of freedom (DOF) can be introduced by the FE