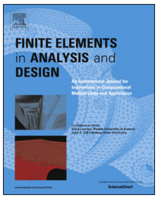
Finite Elements in Analysis and Design 102-103 (2015) 52–64

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel



An extended finite element method for pipe-embedded plane thermal analysis

Zheng Zuo, Yu Hu*, Qingbin Li, Guowei Liu

State Key Laboratory of Hydroscience and Engineering, Department of Hydraulic Engineering, Tsinghua University, 100084 Beijing, China

ARTICLE INFO

Article history:
Received 23 July 2014
Received in revised form
1 May 2015
Accepted 4 May 2015
Available online 22 May 2015

Keywords:
Thermal analysis
Extended finite element
Cooling pipe
Enrichment shape function

ABSTRACT

In thermal analysis of concrete with cooling pipes by conventional FEM (finite element method), extremely refined conforming meshes have to be generated around the pipe axis to obtain precise temperature field. It makes the meshing process demanding and cumbersome, especially in 3D cases. To avoid the difficulty of refining meshes in conventional FEM, an extended finite element method (XFEM) is proposed in this paper with which thermal analysis with pipes can be performed on coarse meshes instead of refined conforming meshes. The enrichment shape function is constructed in the proposed XFEM based on analytical solution and the Dirichlet boundary condition of the cooling pipe is numerically approximated by in-element Robin boundary condition. Thermal analysis of a concrete slab with a cooling pipe in the center is performed by XFEM with coarse meshes and by conventional FEM with refined conforming meshes respectively. The results demonstrate that the temperature field obtained by XFEM has a comparable accuracy level with the conventional FEM. The accuracy and reliability are further validated with three other numerical cases including the varying cooling boundary case, the internal heat source case and the multiple-pipe case.

1. Introduction

Multi-scale problems are always involved in every aspect of engineering field. Heat exchanger with water-circulating pipes embedded in large-scale structures is one of the application cases. They are widely utilized in the field of thermal engineering, civil engineering and mechanical engineering, such as water cooling pipe in computing clusters [1,2], pipe-cooler in nuclear power plant [3], heat pump embedded underground [4,5], pipe-heating system in buildings [6] and so on. One significant case is the cooling pipes used in mass concrete engineering [7–14], which is the main motivation of this work. Concrete releases considerable heat due to the hydration reaction of the cement, and causes temperature gradient near the air-side surface. The temperature gradients are the main causes of thermal cracks that can seriously damage the structure. Active pipe cooling is the most widely applied method to reduce temperature gradient and consequently to avoid or diminish thermal cracks.

Finite element method (FEM) is the most applied numerical approach for thermal analysis of cooling pipe embedded problems because of its quick and stable benchmark. The pipe diameter is relatively smaller than concrete size and sharp temperature gradient is generated around the pipe in the cross plane normal to the pipe axis. In conventional finite element analysis, extremely refined meshes have to be generated around the pipe axis to obtain precise thermal field. Numerous numerical studies have been conducted to analyze the temperature field in mass concrete embedded with cooling pipes [15–24]. Among them, Zhu was the first to systematically propose a finite element flow for the pipe-cooling problem with a refined network of FEM meshes near the pipe [15]. Kim developed a 3D program for the thermal analysis of pipe-cooling systems by marking the pipes as line elements connected by FEM nodes [18]. Liu proposed a heat-fluid coupling method in his work and the cooling pipe is simulated by several line elements chained together considering the heat flow of water [19].

All the above-mentioned FEM simulations are based on refined meshes conforming to the pipe geometry. However, the huge difference between the diameter (about $20-40$ mm) and the length (about $1-2$ m) of the pipe makes the discretization of the domain demanding and cumbersome because the meshes have to conform to the pipe geometry with acceptable aspect ratios. Besides for a 3D problem, the pipes are embedded in grid-like coils, making the meshing procedure more difficult, especially when the structure is large and complex-shaped such as arch dams [25]. Moreover, there are many cases in which multiple pipes

---

* Corresponding author. Tel.: +86 13691000865.
E-mail address: yu-hu@tsinghua.edu.cn (Y. Hu).

http://dx.doi.org/10.1016/j.finel.2015.05.002
0168-874X/© 2015 Elsevier B.V. All rights reserved.

Nomenclature

T	temperature (°C)
ρ	density (kg/m^3)
Γ_p	boundary wall of cooling pipe
T_w	water temperature (°C)
$\boldsymbol{\Psi}$	approximation space
χ_0	maximum adiabatic temperature rise (°C)
r	radial distance (m)
r_0	pipe radius (m)
r_b	outer radius of concrete cylinder (m)
J	Bessel function of first kind
$\Delta\tau$	time increment (day)
$\boldsymbol{H}$	stiffness matrix of heat transfer
$\boldsymbol{Q}$	thermal load matrix
e	Element
c	specific heat capacity (kJ/kg °C)
λ	thermal conductivity (kJ/m °C day)
Ω	concrete domain
Γ_β	equivalent boundary
N	conventional Lagrangian shape function
ϕ	enriched shape function
m	parameter representing the heat generation rate
β	convective coefficient (kJ/m^2 °C day)
θ	excess temperature (°C)
Y	Bessel function of second kind
ε	characteristic time factor (day)
T_0	initial temperature (°C)
$\boldsymbol{P}$	mass matrix of heat transfer
τ	time (d)

are used, making the analysis process very costly in time and computable resources.

In this paper, an extended finite element method (XFEM) [26–28] is proposed to simulate the thermal field around the cooling pipes using coarse meshes, avoiding the difficulty of generating refined conforming meshes. Advantages of the proposed XFEM is that it could make the numerical meshes independent from the domain to be discretized, which is achieved by integration of a priori knowledge of the solution field and the enrichment shape functions. The paper is organized as follows. Firstly, the governing equation of the problem is given in Section 2. The XFEM formulation is introduced to approximate the discontinuous thermal gradient field in Section 3. Secondly, the enrichment shape function is constructed on the basis of the analytical solution in Section 4. The cooling-pipe boundary condition and the quadrature scheme for discontinuous element are also discussed in details in this section. Finally, a series of numerical cases are carried out in Section 5 in which the validity and efficiency of the method are shown.

2. Problem description

The temperature variation along the axial direction of the pipe is considerably smaller than that in the radial direction. Thus this study focuses on the 2D cooling problem in the cross section normal to the pipe axis, as shown in Fig. 1. The governing equation of the problem can be expressed as follows:

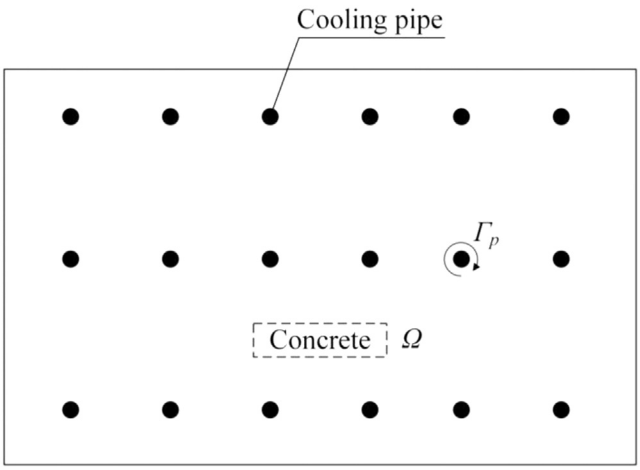


Fig. 1. 2D schematic diagram of concrete domain and cooling pipe, in which the pipe is perpendicular to the plane.

$$\rho c\frac{\partial T}{\partial \tau}=\nabla(\boldsymbol{\lambda}\nabla T)+Q(\tau) \quad \text{in } \Omega, \tag{1}$$

where T is the temperature scalar, τ is the time, ρ is the density of concrete, c is the specific heat capacity of concrete, $Q(\tau)$ is the internal heat source of concrete, and $\boldsymbol{\lambda}$ is the heat conductivity tensor, which is considered isotropic in this study and denoted as a scalar λ.

Metal pipes are always applied in thermal engineering for their good heat conductivity. Dirichlet boundary condition, which is also known as the first type boundary condition is imposed on the pipe wall,

$$T=T_w(\tau) \quad \text{on } \Gamma_p, \tag{2}$$

where T_w is the water temperature and Γ_p is the wall of the cooling pipe.

The internal heat source of concrete is induced by the hydration heat, which can be expressed as

$$Q(\tau)=\rho c\chi(\tau), \tag{3}$$

where $\chi(\tau)$ is the adiabatic temperature rise of concrete. The hydration heat in concrete increases very quickly at the early age, and then decreases gradually with the age, which is always expressed in an exponential form [29]:

$$\chi(\tau)=\chi_0\left(1-e^{-m\tau}\right), \tag{4}$$

where χ_0 is the maximum temperature rise of concrete under adiabatic condition, m is the parameter that represents the heat generation rate.

Initially, we suppose that the water is not running in the cooling pipes. Therefore, the temperature field is uniform on the concrete domain, which provides the following:

$$T=T_0(x) \quad \text{when } \tau=0, \tag{5}$$

where $T_0(x)$ is the initial temperature of the concrete.

3. XFEM formulation

3.1. Approximating thermal field

Generally the temperature field is approximated by the conventional FEM, as shown in the following equation:

$$T_{appx}(x)=\sum_{i=1}^{i\in S}N_i(x)T_i. \tag{6}$$