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A successive perturbation-based multiscale stochastic analysis method for composite materials

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ABSTRACT

This paper introduces a successive perturbation-based method for multiscale stochastic analysis of heterogeneous materials such as composite materials. Microscopic random variations sometimes have a significant influence on homogenized material properties and microscopic stress fields. Therefore, multiscale stochastic problems should be analyzed to evaluate the reliability of composite structures. Further, in order to assess the result of numerical analysis of a composite structure, this type of uncertainty propagation should be taken into account. For this purpose, the successive perturbation based approach is proposed.

As a numerical example, the stochastic homogenization and multiscale stochastic stress analysis problem of composite materials are solved considering microscopic random variations. From the numerical results, the proposed approach gives a more accurate estimation than the conventional perturbation approach. Also, our proposed method works well for both smooth nonlinear response functions and non-smooth response functions.

1. Introduction

Estimating the influence of microscopic random variations of heterogeneous materials such as composite materials on a homogenized material property is called as the stochastic homogenization problem. Kaminski et al. [1] and Sakata et al. [2] reported stochastic homogenization analysis with the Monte-Carlo simulation. For single-scale stochastic problems, the stochastic finite element method [3] and the spectral stochastic finite element method (SSFEM) [4] have been proposed, and they have been extended to the multiscale stochastic analysis for heterogeneous materials such as composites in recent. For example, the perturbation-based approach for elastic [5–7] and thermal problems [8–10] and SSFEM based multiscale stochastic analysis [11] have been reported. This research topic is noticed recently, and other approaches have been also reported for stochastic homogenization analysis [12–14].

The stochastic homogenization problem is important for reliability evaluation of composite structures, especially in cases where a priori random variation or a posteriori effects such as aging and mechanical and chemical influences in a manufacturing process should be taken into account. In addition, from the viewpoint of assessment using computer simulation-based analysis, this type of uncertainty propagation through the different scales should be considered in the validation and verification of the numerical simulation.

The randomness of a microscopic field in a heterogeneous material, a key feature of this paper, influences the structural response to be evaluated. Because a heterogeneous advanced material has a complex microstructure, multiscale analysis methods that analyze the homogenized material property and multiscale stress field are usually employed. Similarly, influence of the microscopic uncertainty on the property of homogenized materials and the macroscopic and microscopic stress field should be analyzed with a multiscale analysis framework.

In general cases, the Monte-Carlo method gives good estimation of the probabilistic response with less assumption, but a computational cost will be expensive. Thus, a more accurate and efficient stochastic homogenization methodology is needed. Kaminski [15] and Sakata et al. [7,10] reported numerical results by employing a higher order perturbation-based method; however, it has been reported that higher order perturbation does not always improve estimation accuracy. Approximation-based stochastic homogenization methodologies have been proposed to solve this problem by Sakata et al. [16] and Kaminski [17]. These approximate stochastic homogenization methods effectively improve accuracy and computational efficiency for the analysis, but problems still exist. For example, determination of the approximation parameters and applicability of those methods to problems having many random variations need to be explored. Thus,

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the series expansion-based approach needs to be improved. Some related topics can be found in literature [18].

In this paper, the perturbation-based technique is improved for more accurate multiscale stochastic analysis. A higher order perturbation does not always improve estimation accuracy, and it is difficult to determine an appropriate approximation order for acceptable estimation without referring to an exact solution of the probabilistic property obtained from experimental results or Monte-Carlo simulation; therefore, a lower order approximation is preferable in some cases. Also, a general perturbation method with using derivatives at single expansion point or a smooth function approximation will not be applicable to a non-smooth stochastic response such as the maximum microscopic stress against a microscopic random variation. Consequently, a successive perturbation-based multiscale analysis method is proposed in this paper.

In the following sections, at first, an outline of the stochastic homogenization problem and the multiscale stochastic stress analysis methodology is introduced in Section 2. Next, details of the proposed method are given in Section 3. The proposed method is applied to the stochastic homogenization problem on thermoelastic properties and multiscale stress analysis of composite materials, and its validity and effectiveness and the numerical results obtained are discussed in Sections 4, 5, and 6. Finally, conclusions are given in Section 7.

2. Multiscale stochastic analysis in elastic problems

Multiscale analysis is important in the design of composite structures because composite materials possess complex microstructures that exert complex influences on macroscopic properties and the microscopic stress fields of composite materials. In general, a structure is divided into two or more scales: one is a macroscopic structure, and the other is a microscopic structure comprising two or more component materials. The equivalent material property that reflects the features of the microscopic structure, such as the material properties of the geometry of the component materials, is used in mechanical analysis at the macroscale, and therefore multiscale stress analysis involves the procedures of homogenization, macroscopic stress analysis, and localization. An outline of deterministic multiscale elastic analysis is illustrated in Fig. 1(a).

In addition, the influence of microscopic randomness on macroscopic response and microscopic stress fields has been analyzed. This type of analysis is an important issue in solid mechanics, especially in the case of heterogeneous materials. In such cases, propagation of uncertainty or randomness through the different scales must be investigated with stochastic homogenization or multiscale stochastic stress analysis, which investigates the stochastic response caused from a random variation in a different scale, especially in microscale. An outline of multiscale stochastic analysis is illustrated in Fig. 1(b). In this paper, the multiscale stochastic problem is analyzed.

In general, there are two types of microscopic random variations; one is the uniformly distributed random variation, which is observed in material properties of component materials in different production lots, the other is non-uniform random variation, which can be observed in a random media. Both cases should be considered in reliability evaluation of composite structures, and the former case is considered in this paper.

3. Analysis method

3.1. Perturbation-based stochastic homogenization method

From a general formulation of homogenization theory [19], a homogenized macroscopic elastic tensor $\boldsymbol{E}^H$ can be computed as

$$\boldsymbol{E}^H = \frac{1}{|\mathbf{Y}|}\int_Y \boldsymbol{E}\left(\boldsymbol{I} - \frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{y}}\right)\mathrm{d}Y, \tag{1}$$

where $\boldsymbol{E}$ is an elastic tensor of the microstructure, $|\boldsymbol{Y}|$ is the volume of a unit cell, and $\boldsymbol{I}$ is a unit tensor. χ is a characteristic displacement, which can be obtained as a solution of the following characteristic equation:

$$\int_Y \frac{\partial}{\partial \boldsymbol{y}}\boldsymbol{E}\frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{y}}\mathrm{d}Y = \int_Y \frac{\partial}{\partial \boldsymbol{y}}\boldsymbol{E}\mathrm{d}Y \tag{2}$$

If a microscopic quantity such as an elastic property of a component material includes a random variation, the homogenized elastic tensor can be expressed as a stochastic response for the microscopic random variable as follows:

$$\boldsymbol{E}^{H*} = \frac{1}{|\mathbf{Y}|}\int_Y \boldsymbol{E}^*\left(\mathbf{I} - \frac{\partial \boldsymbol{\chi}^*}{\partial \boldsymbol{y}}\right)\mathrm{d}Y \tag{3}$$

where the superscript "*" indicates a random variable. In addition, the characteristic equation considering the randomness can be rewritten as

$$\int_Y \frac{\partial}{\partial \boldsymbol{y}}\boldsymbol{E}^*\frac{\partial \boldsymbol{\chi}^*}{\partial \boldsymbol{y}}\mathrm{d}Y = \int_Y \frac{\partial}{\partial \boldsymbol{y}}\boldsymbol{E}^* dY \tag{4}$$

The probabilistic characteristics as the expectation or variance of the homogenized elastic tensor can be expressed as

$$\mathrm{Exp}\left[\boldsymbol{E}^{H*}\right] = \int_{-\infty}^{\infty} \boldsymbol{E}^H f\left(\boldsymbol{E}^H\right)\mathrm{d}\boldsymbol{E}^H \tag{5}$$

$$\mathrm{Var}\left[\boldsymbol{E}^{H*}\right] = \int_{-\infty}^{\infty} \left(\boldsymbol{E}^H - \mathrm{Exp}\left[\boldsymbol{E}^{H*}\right]\right)^2 f\left(\boldsymbol{E}^H\right)\mathrm{d}\boldsymbol{E}^H \tag{6}$$

where $\mathrm{Exp}\left[\boldsymbol{E}^{H*}\right]$ and $\mathrm{Var}\left[\boldsymbol{E}^{H*}\right]$ are the expectation and variance of the homogenized elastic tensor, and $f\left(E^H\right)$ is the probabilistic density function of the homogenized elastic tensor.

If the stochastic homogenized elastic tensor can be expressed with a series expansion-based approximation, e.g.,

$$\boldsymbol{E}^{H*} \approx \sum_i \boldsymbol{E}^{Hi}\varphi_i, \tag{7}$$

then the probabilistic characteristic can be estimated with an approximation form as

$$\mathrm{E}\left[\boldsymbol{E}^{H*}\right] = \int_{-\infty}^{\infty} f\left(\boldsymbol{E}^H\right)\sum_i \boldsymbol{E}^{Hi}\varphi_i \mathrm{d}\boldsymbol{E}^H \tag{8}$$

$$\mathrm{Var}\left[\boldsymbol{E}^{H*}\right] = \int_{-\infty}^{\infty} f\left(\boldsymbol{E}^H\right)\left(\sum_i \boldsymbol{E}^{Hi}\varphi_i - \mathrm{Exp}\left[\boldsymbol{E}^{H*}\right]\right)^2 \mathrm{d}\boldsymbol{E}^H \tag{9}$$

where φ_i is an arbitrary basis function for the approximation of the stochastic homogenized elastic tensor, and $\boldsymbol{E}^{Hi}$ is a coefficient for each term of the basis function.

If the random variation of a microscopic quantity, for example, Young's modulus of a component material E^*, can be expressed with its expected value and a normalized random variable β as $E^* = E^0(1+\beta)$, and it is assumed that the basis function can be expressed by the power series as

$$\varphi_i = \beta^i, \tag{10}$$

then, the approximated form of the homogenized elastic tensor can be written as

$$\boldsymbol{E}^{H*} \approx \sum_i \boldsymbol{E}^{Hi}\beta^i \tag{11}$$

where $\mathbf{E}^{Hi}$ is called the *ith* order perturbation term or derivative.

From the perturbation-based stochastic homogenization theory, the perturbation term of the homogenized elastic tensor is computed at relatively lower computational costs.

For instance, when the microscopic elastic tensor and the characteristic displacement vector are expressed by the asymptotic expansion with respect to the random variable β, the observed value of the homogenized elastic tensor can be approximately