Finite Elements in Analysis and Design 64 (2013) 79–89

Contents lists available at SciVerse ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

A rotation-free beam element for beam and cable analyses

Y.X. Zhou, K.Y. Sze*

Department of Mechanical Engineering, The University of Hong Kong, Pokfulam, Hong Kong

A R T I C L E I N F O

Article history:
Received 28 August 2012
Accepted 4 October 2012
Available online 1 November 2012

Keywords:
Rotation- free
Beam
Finite element
Corotational
Geometric nonlinear
Cable

A B S T R A C T

The rotation-free or RF element method represents a non-conventional finite element method in which the rotations are not used as dofs and the element interpolation domains are overlapping. Its obvious advantage is that the complication of finite rotations can be avoided. In this paper, the relatively unexplored RF plane beam element recently formulated by the authors in the course of developing a RF triangle for thin-shell analyses is revisited. Comparing with other RF plane beam elements, the present one is simple and physical yet its accuracy remains competitive. Using a corotational approach and the small strain assumption, its tangent bending stiffness matrix can be approximated by a constant matrix which does not require updating in geometric nonlinear analyses. The element is here extended to spatial cable analyses in which the torsional stiffness can often be neglected and the sectional properties are isotropic. Under the same nodal distributions, it is seen that the present element can tolerate much larger load increment and time step under static and dynamic analyses, respectively, than the two-node thin beam finite element.

1. Introduction

Rotation-free or RF element method has attracted considerable attention in the last two decades. An obvious advantage of the method is that it simplifies the kinematic description under finite rotations. While the focus of the method is on RF triangular plate/shell elements [1–7], the RF beam elements are relatively unexplored until more recently. In the RF element, its interpolation domain is larger than its integration domain which is referred to as "element" for simplicity. In other words, not only the nodes within but also adjacent to the element are employed in the displacement interpolation. Phaal and Calladina [1] developed a RF beam element based on the quadratic interpolation. Three nodes are used to construct the displacement from which a constant curvature can be derived, see Fig. 1(a). To the best knowledge of the authors, this straight forward linear straight beam formulation was not extended to curved beam and nonlinear analyses. Flores and Oñate [8] presented RF elements for nonlinear analyses of plane beams and axisymmetric shells with special emphasis on treating non-smooth and branching beams. With respect to Figs. 1(b), 1–2, 2–3 and 3–4 are treated as straight and their directors ($\mathbf{n}_{12}$, $\mathbf{n}_{23}$ and $\mathbf{n}_{34}$) after deformation are computed accordingly in Ref. [8]. Based on the displaced directors, the curvatures at nodes 2 and 3 are determined and linearly interpolated for the element bounded by the two nodes. Oñate and Zarate [9] later included the transverse shear deformation into the formulation by introducing shear angle dofs. On the other hand, Battini [10] proposed a RF corotational plane beam element. The element again relies on four nodes and the corotational frame is aligned with nodes 2 and 3. Using nodes 1 to 4 and nodes 1 to 3, cubic and quadratic local deflections are interpolated, respectively. The local rotation at node 2 is taken as the average rotations derived from the two deflections. Similarly, the local rotation at node 3 can be derived. Using the local rotations and the zero local deflections at the two nodes, another local cubic transverse deflection is derived for the element bounded by nodes 2 and 3.

Very recently, the authors have formulated a RF plane beam element in the course of developing a RF triangle plate/shell element [11]. The formulation can be regarded as an extension of the simple RF beam element of Phaal and Calladina [1] to the curved beam and geometric nonlinear analyses. Comparing with other RF beam elements, the present one is simple and physical but its accuracy is competitive. Using a corotational approach and the small strain assumption, its tangent bending stiffness matrix can be approximated as a constant matrix which does not require updating in a geometrically nonlinear analysis. It is particularly suitable for efficient analysis of highly geometrically nonlinear problems. Cables, which are used in cable-supported bridges and roofs, are typical examples [12].

In computational analysis of cables, linear and higher order line finite elements [13,14], catenary finite elements [12,15–18] and, of course, beam finite elements can be employed. As the catenary elements combine the analytical catenary expressions

* Corresponding author. Tel.: +852 2859 2637; fax: +852 2858 5415.
E-mail address: kysze@hku.hk (K.Y. Sze).

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http://dx.doi.org/10.1016/j.finel.2012.10.002

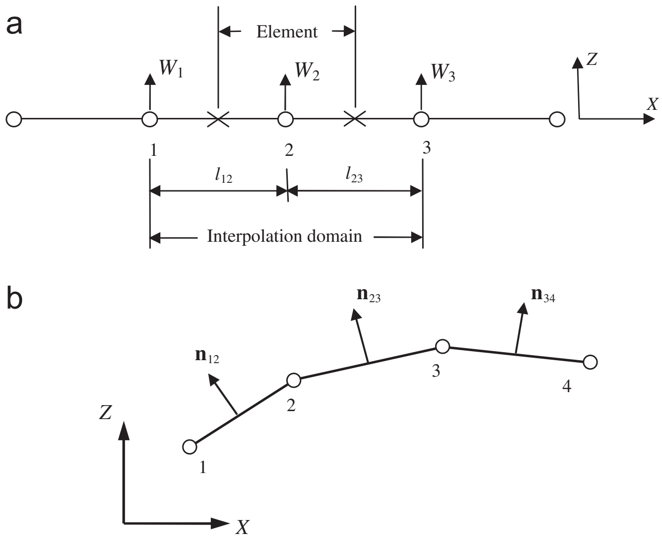

Fig. 1. (a) Three consecutive nodes along a straight plane beam. (b) Four consecutive nodes along a curved plane beam.

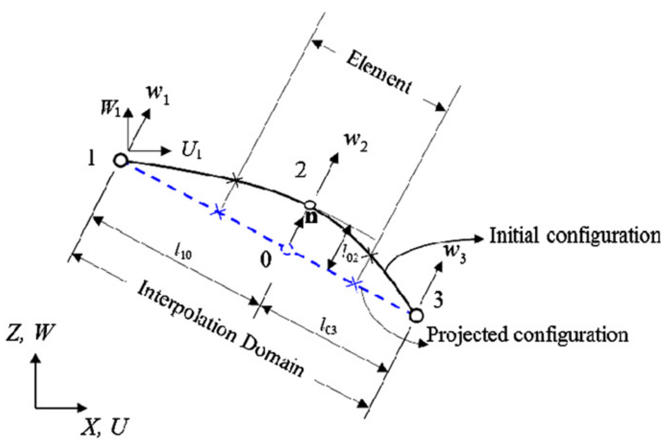

Fig. 2. Three consecutive nodes along a curved plane beam.

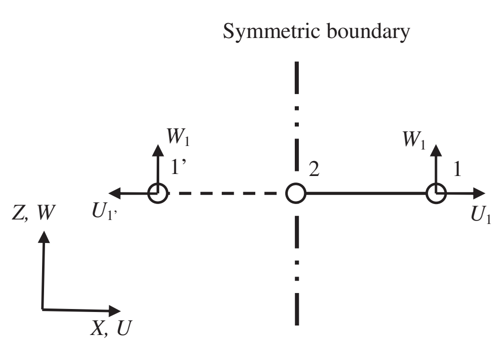

Fig. 3. Imposition of symmetric boundary condition.

with the numerical method, they can yield accurate static predictions by using very few elements.

Nevertheless, there are a couple of drawbacks in the line and catenary elements. Firstly, they concern only the axial force and the bending effect is ignored. This is justifiable in most but not all cases. For example, Irvin [19] pointed out that when rapid changes in curvature are unavoidable, the bending effect may be locally important. Recently, Buckham et al. [20] also indicated that the bending effect is sometimes important, e.g. in the dynamic

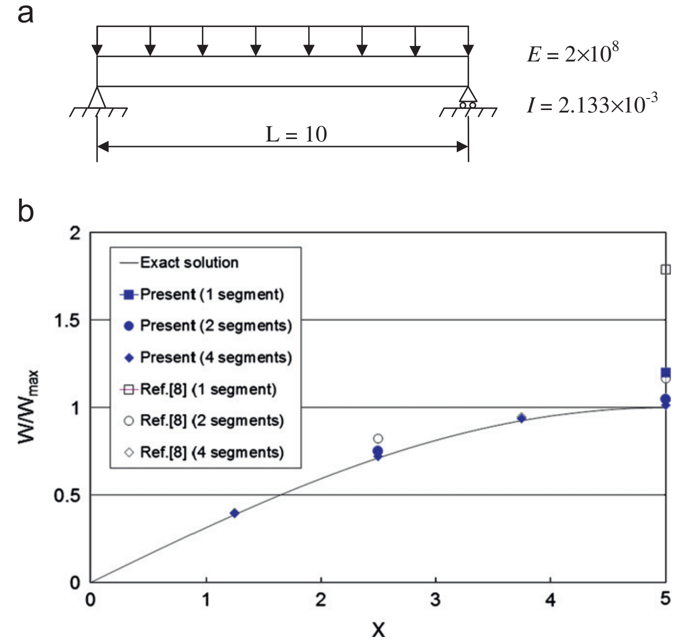

Fig. 4. (a) Simple supported plane beam under uniform load. (b) Normalized deflections along the beam.

simulation of slack tethers used in underwater remotely operated vehicles. Secondly, numerical instability and convergence difficulties are sometimes encountered. Thus, some additional schemes such as pre-stress, pre-strain, assumed configuration and form-finding have been proposed [21]. Nevertheless, proper choice of these schemes and the related settings are not straight forward. Of course, these drawbacks do not exist in the beam finite element. Recently, a 'nodal coordinate element' [22,23] has been proposed to deal with aforementioned drawbacks. The element takes bending and transverse shear effects into account. However, in their formulation, not only the nodal coordinates but also the slopes of the coordinates need to be taken as the nodal dofs. For 3D analysis, each node carries 12 dofs. The formulation is complicated and the computational effort is considerably large.

In the present paper, the RF beam element proposed in Ref. [11] will be re-visited and applied to the cable applications. The outline of this paper is as follows. The linear formulation of the beam element is reviewed in Section 2 followed by some numerical examples in Section 3. In Section 4, the corotational approach is employed to extend linear element to geometrically nonlinear analyses. Nonlinear numerical examples are given in Section 5. It should be remarked that the materials presented in Sections 2 and 4 have been similarly presented in Ref. [11]. However, Ref. [11] covers linear and nonlinear straight beams, curved beams, plates and shells. Consequently, only two smooth and relatively unconstrained beam examples are presented. The examples presented here are markedly different in nature from the two in Ref. [11]. They include constrained and folded beams which have also been considered by other RF beams. Our RB beam is indeed comparable to those published by the others in accuracy yet its formulation is much simpler. In Section 6, a pseudo 3D RF beam element is newly developed and employed in cable analyses as presented in Section 7. Under the same nodal distributions, it can be seen that the present element (with only translational dofs) can tolerate much larger load increment and time step under static and dynamic analyses, respectively, than the two-node thin beam finite element (with translational and rotational dofs).