Finite Elements in Analysis and Design 63 (2013) 42–50

Contents lists available at SciVerse ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Dynamic finite element analysis of axially vibrating nonlocal rods

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ARTICLE INFO

Article history:
Received 9 March 2012
Received in revised form
31 May 2012
Accepted 7 August 2012
Available online 24 September 2012

Keywords:
Axial vibration
Nonlocal mechanics
Dynamic stiffness
Asymptotic analysis
Frequency response

ABSTRACT

Free and forced axial vibrations of damped nonlocal rods are investigated. Two types of nonlocal damping models, namely, strain-rate-dependent viscous damping and velocity-dependent viscous damping, are considered. A frequency-dependent dynamic finite element method is developed to obtain the forced vibration response. Frequency-adaptive complex-valued shape functions are proposed to obtain the dynamic stiffness matrix in closed form. The stiffness and mass matrices of the nonlocal rod are also obtained using the conventional finite element method. Results from the dynamic finite element method and conventional finite element method are compared. Using an asymptotic analysis it is shown that, unlike its local counterpart, a nonlocal rod has a maximum cut-off frequency. A closed-form exact expression for this maximum frequency as a function of the nonlocal parameter has been obtained for undamped and damped systems. The frequency response function obtained using the proposed dynamic finite element method shows extremely high modal density near the maximum frequency. This leads to clustering of resonance peaks which is not easily obtainable using classical finite element analysis.

1. Introduction

Research on size-dependent structural theories for the accurate design and analysis of micro and nanostructures is growing rapidly [1–4]. This is because, though molecular dynamic (MD) simulation is justified for the analysis of nanostructures [5,6] such as nanorods, nanobeam, nanoplates, nanoshells and nanocones, the approach is computationally exorbitant for nanostructures with large numbers of atoms. This calls for the use of conventional continuum mechanics [7] and finite elements in analysis of nanostructures. However, classical continuum modelling approach is considered scale-free. It fails to account for the effects arising from the small-scale where size-effects are prominent.

Nanoscale experiments demonstrate that the mechanical properties of nano dimensional materials are much influenced by size effects or scale effects [8,9]. Size effects are related to atoms and molecules that constitute the materials. Further, atomistic simulations have also reported size effects on the magnitudes of resonance frequency and buckling load of nanoscale objects such as nanotubes and graphene [10,11]. The application of classical continuum approaches is thus questionable in the analysis of nanostructures such as nanorods, nanobeams and nanoplates. Examples of nanorods and nanobeams include carbon and boron nanotubes, while nanoplates can be graphene sheets or gold nanoplates.

One widely promising size-dependant continuum theory is the nonlocal elasticity theory pioneered in [12] which brings in the scale effects and underlying physics within the formulation. Nonlocal elasticity theory contains information related to the forces between atoms, and the internal length scale in structural, thermal and mechanical analyses. In the nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain. Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body [12]. Some drawbacks of classical continuum theory can be efficiently avoided and the size-dependent phenomena can be reasonably explained by nonlocal elasticity. Recent literature shows that the theory of nonlocal elasticity is being increasingly used for reliable and fast analysis of nanostructures. Studies include nonlocal analysis of nanostructures viz. nanobeams [13–15], nanoplates [16], carbon nanotubes [17], graphene [18], microtubules [19] and nanorings [20].

Recently due to elevated interests in nanotechnology, various one-dimensional nanostructures have been realised. They include nanorods, nanowires, nanobelts, nanotubes, nanobridges, nanonails, nanowalls and nanohelixes. Among all the one-dimensional nanostructures, nanotubes, nanorods and nanowires are the most widely studied. This is because of the easy material formation and device applications. One important one-dimensional

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http://dx.doi.org/10.1016/j.finel.2012.08.001

nanostructure is nanorods. Nanorods [21] are one-dimensional objects ranging from 1 to 3000 nm in length. They can be grown from various methods, including (i) vapour phase synthesis [22], (ii) metal-organic chemical vapour deposition [23], (iii) hydrothermal synthesis [24]. Nanorods have found application in a variety of nanodevices, including ultraviolet photodetectors, nanosensors, transistors, diodes and LED arrays.

Axial vibration experiments can be used for the determination of the Youngs modulus of Carbon Nanotubes (CNTs). Generally, the flexural modes occur at low frequencies. However vibrating nanobeams (CNTs) may also have longitudinal modes at relatively high frequencies and can be of very practical significance in high operating frequencies. Nanorods when used as electromechanical resonators can be externally excited and exhibit axial vibrations. Furthermore for a moving nanoparticle inside a single-walled carbon nanotube (SWCNT), the SWCNT generally vibrates both in the transverse and longitudinal directions. The longitudinal vibration is generated because of the friction existing between the outer surface of the moving nanoparticle and the inner surface of the SWCNT. It is also reported [25] that transport measurements on suspended SWCNTs show signatures of phonon-assisted tunnelling, influenced by longitudinal vibration (stretching) modes. Chowdhury et al. [26] have reported sliding axial modes for multiwalled carbon nanotubes (MWCNTs). Tong et. al [27] have considered axial buckling of MWCNTs with heterogeneous boundary conditions.

Only limited work on nonlocal elasticity has been devoted to the axial vibration of nanorods. Aydogdu [28]developed a nonlocal elastic rod model and applied it to investigate the small scale effect on the axial vibration of clamped-clamped and clamped-free nanorods. Filiz and Aydogdu [29] applied the axial vibration of nonlocal rod theory to carbon nanotube heterojunction systems. Narendra and Gopalkrishnan [30] have studied the wave propagation of nonlocal nanorods. Recently Murmu and Adhikari [31] have studied the axial vibration analysis of a double-nanorod-system. In this paper, we will be referring to a nanorod as a nonlocal rod, so as to distinguish it from a local rod.

Several computational techniques have been used for solving the nonlocal governing differential equations. These techniques include Naviers Method [32], Differential Quadrature Method (DQM) [33] and the Galerkin technique [34]. Recently attempts have been made to develop a Finite Element Method (FEM) based on nonlocal elasticity. The upgraded finite element method in contrast to other methods above can effectively handle more complex geometry, material properties as well as boundary and/or loading conditions. Pisano et al. [35] reported a finite element procedure for nonlocal integral elasticity. Recently some motivating work on a finite element approach based on nonlocal elasticity was reported [36]. The majority of the reported works consider free vibration studies where the effect of non-locality on the eigensolutions has been studied. However, forced vibration response analysis of nonlocal systems has received very little attention.

Based on the above discussion, in this paper we develop the dynamic finite element method based on nonlocal elasticity with the aim of considering dynamic response analysis. The dynamic finite element method belongs to the general class of spectral methods for linear dynamical systems [37]. This approach, or approaches very similar to this, is known by various names such as the dynamic stiffness method [38–48], spectral finite element method [37,49] and dynamic finite element method [50,51]. Some of the key features of the method are:

- The mass distribution of the element is treated in an exact manner in deriving the element dynamic stiffness matrix.
- The dynamic stiffness matrix of one-dimensional structural elements, taking into account the effects of flexure, torsion, axial and shear deformation, and damping, is exactly determinable, which, in turn, enables the exact vibration analysis by an inversion of the global dynamic stiffness matrix.
- The method does not employ eigenfunction expansions and, consequently, a major step of the traditional finite element analysis, namely, the determination of natural frequencies and mode shapes, is eliminated which automatically avoids the errors due to series truncation.
- Since modal expansion is not employed, ad hoc assumptions concerning the damping matrix being proportional to the mass and/or stiffness are not necessary.
- The method is essentially a frequency-domain approach suitable for steady state harmonic or stationary random excitation problems.
- The static stiffness matrix and the consistent mass matrix appear as the first two terms in the Taylor expansion of the dynamic stiffness matrix in the frequency parameter.

So far the dynamic finite element method has been applied to classical local systems only. In this paper we generalise this approach to nonlocal systems. One of the novel features of the analysis proposed here is the employment of frequency-dependent complex nonlocal shape functions for damped systems. This in turn enables us to obtain the element stiffness matrix using the usual weak form of the finite element method.

The paper is organised as follows. In Section 2 we introduce the equation of motion of axial vibration of undamped and damped rods. Natural frequencies and their asymptotic behaviours for both cases are discussed for different boundary conditions. The conventional and the dynamic finite element method are developed in Section 3. Closed form expressions are derived for the mass and stiffness matrices. In Section 4 the proposed methodology is applied to an armchair single walled carbon nanotube (SWCNT) for illustration. Theoretical results, including the asymptotic behaviours of the natural frequencies, are numerically illustrated. Finally, in Section 5 some conclusions are drawn based on the results obtained in the paper.

2. Axial vibration of damped nonlocal rods

2.1. Equation of motion

The equation of motion of axial vibration for a damped nonlocal rod can be expressed as

$$\begin{aligned}EA\frac{\partial^2 U(x,t)}{\partial x^2}+\widehat{c}_1\left(1-(e_0a)_1^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^3 U(x,t)}{\partial x^2\partial t}\\ =\widehat{c}_2\left(1-(e_0a)_2^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial U(x,t)}{\partial t}+\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\\ \times\left\{m\frac{\partial^2 U(x,t)}{\partial t^2}+F(x,t)\right\}\end{aligned} \quad (1)$$

This is an extension of the equation of motion of an undamped nonlocal rod for axial vibration [28,31,52]. Here EA is the axial rigidity, m is the mass per unit length, e_0a is the nonlocal parameter [12], $U(x,t)$ is the axial displacement, $F(x,t)$ is the applied force, x is the spatial variable and t is the time. The constant $\widehat{c}_1$ is the strain-rate-dependent viscous damping coefficient and $\widehat{c}_2$ is the velocity-dependent viscous damping coefficient. The parameters $(e_0a)_1$ and $(e_0a)_2$ are nonlocal parameters related to the two damping terms respectively. For simplicity we have not taken into account any nonlocal effect related to the damping. Although this can be mathematically incorporated in the analysis, the determination of these nonlocal parameters is beyond the scope of this work and therefore only local interaction for the damping is adopted. Thus, in the following analysis we