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# Heat transfer element for modeling the thermal response of non-uniformly heated plates

Ann E. Jeffers *

*Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI, USA*

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ABSTRACT

This paper presents a novel heat transfer element for modeling the 3D thermal response in plates and shells exposed to non-uniform heating. The formulation uses a combination of finite element and control volume techniques to discretize the governing equations into a series of 2D layers that are linked by a finite difference calculation. While a combination of techniques are used in the element formulation, the governing element equations are in a form that can readily be implemented into a commercial finite element code. The nine-node quadratic element considered here is implemented in *ABAQUS* as a user-defined element. Two- and three-dimensional verification cases are presented to demonstrate the capabilities of the element. Comparisons between the proposed shell heat transfer element and traditional continuum elements demonstrate that the proposed model exhibits a high level of accuracy and requires minimal computational power. The layered formulation offers the additional advantage of simplifying the transfer of temperature data from a thermal analysis to a structural analysis because both models can have the same mesh.



## 1. Introduction

Structural response in fire is typically modeled by a series of sequentially coupled analyses, as illustrated in Fig. 1. First, a fire analysis is conducted to evaluate the thermal boundary conditions at the surface of the structure. Then, a heat transfer analysis is conducted to determine the temperature field in the structure. Lastly, a structural analysis is carried out to evaluate the mechanical response of the structure based on the degradation of the material properties and thermal expansion.

The current state of practice emphasizes post-flashover compartment fire exposure in which the fire is modeled by a uniform temperature–time curve [1]. Although fully developed compartment fire models are appropriate for many applications, several studies have indicated that the uniform temperature assumption may be invalid in a number of instances [2,3]. To overcome existing limitations, researchers such as Chen et al. [4] have focused on the development of physics-based simulation tools that enable the analyst to couple a high resolution computational fluid dynamics simulation of the fire with a structural solver for performing a coupled thermo–structural analysis. Such methods offer remarkable opportunities for accurately capturing coupled effects that exist at the fire-structure interface, and it is foreseeable that future models may be able to model complex behaviors such as fire propagation due to partial collapse or burn-through in structures.

Despite radical advances in simulation-based research, severe limitations exist that prohibit the translation of novel multiphysics approaches to industry. In particular, computational fluid dynamics simulations require an extremely fine spatial and temporal resolution to accurately capture the "fast" physics that govern fire propagation. At the structural level, however, spatial and temporal resolutions can be significantly relaxed to a degree that may be several orders of magnitude larger than what is needed for fire simulation (Fig. 1). The transfer of data across disparate scales prohibits the realization of a truly coupled fire-structure simulation because adequate homogenization and subcycling algorithms are not presently included in existing finite element codes [5]. The thermo–structural analysis is further complicated by a need for compatibility between meshes in the solid heat transfer and structural simulations. As a result, an analyst must choose between (a) running a high-resolution 3D thermo–structural simulation using computationally expensive continuum elements, or (b) simplifying the thermal response (e.g., conducting a 1D heat transfer analysis over the thickness) such that the structural response can be modeled using more efficient beam or shell elements. The former is overwhelmingly expensive for even a simple plate structure, while the latter may violate the actual heating conditions of the structure.

To overcome this limitation, Jeffers and Sotelino [6] formulated a heat transfer finite element for simulating the 3D thermal response in non-uniformly heated structural frames. The element

* Corresponding author. Tel.: +1 734 763 6832; fax: +1 734 764 4292.
*E-mail address:* jffrs@umich.edu

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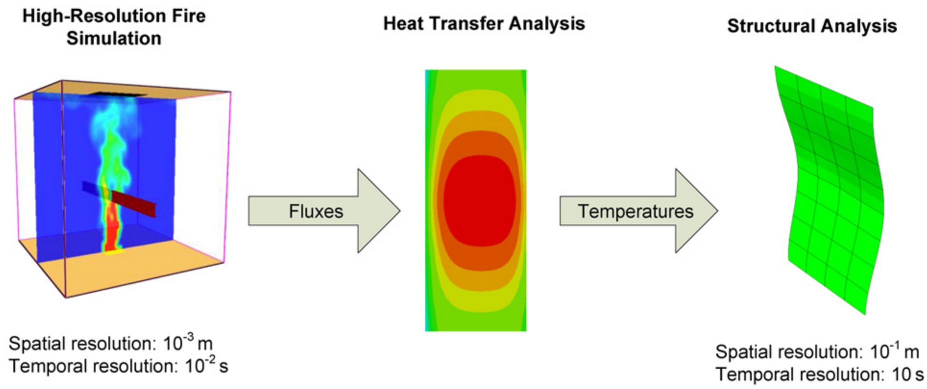


**Fig. 1.** Transfer of thermal data across disparate scales in modeling structures in fire.

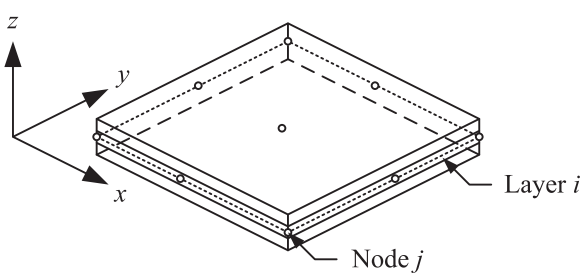


**Fig. 2.** Shell heat transfer element.

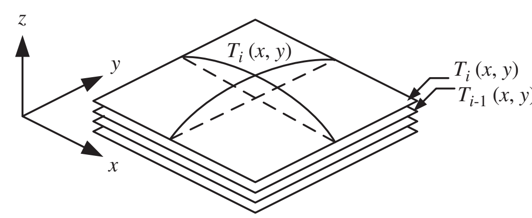


**Fig. 3.** Schematic of the temperature field in layer *i*.

used a combination of finite element and control volume techniques to discretize the temperature field into a series of 1D fibers. The element was shown to provide an accurate prediction of the 3D temperature field in non-uniformly heated frame structures. Furthermore, the fiber discretization simplified the transfer of temperature data from the thermal analysis to the structural analysis because the heat transfer element was compatible with distributed plasticity elements that are often used in the non-linear analysis of structural frames [7].

The present study extends the work of Jeffers and Sotelino [6] to the analysis of plate and shell structures. In particular, a novel heat transfer element is presented here to capture the 3D thermal response in non-uniformly heated plates and shells. The proposed element is similar in spirit to a class of elements proposed by Prof. Surana's group [8–12], but the present formulation has several distinct characteristics in the formulation and in the capabilities of the element. Prior heat transfer elements for the analysis of laminated shells have focused on a hierarchical *p*-version formulation, in which the temperature field is interpolated from nodal values in the three coordinate directions, with continuity conditions imposed at the interlaminar boundaries to allow the temperature field in the transverse direction to be approximated as a piecewise function. The lowest degree of polynomial that can be represented in the *p*-version formulation is a linear function, which requires two nodes for each layer in the shell.

In the present formulation, a combination of finite element and control volume techniques are used to discretize the temperature field into a series of 2D layers, as shown in Fig. 2, that are each characterized by a single (i.e., constant) temperature in the transverse direction. The layering approach allows the temperature field to be modeled in high resolution over the depth of the element whereas temperatures in plane are approximated by linear or quadratic interpolation functions. The lumped mass approximation in the transverse direction results in an element that is highly efficient and compatible with shell elements that are widely used in structural mechanics [13–15], which use layering to simulate the spread of plasticity and to account for variations in material properties over the depth. The proposed heat transfer element provides a one-to-one mapping between the temperature degrees of freedom in the heat transfer element and the points of integration in the structural element (i.e., for the calculation of thermal strains and the application of temperature-dependent constitutive relationships), without the need for additional calculations to interpolate and/or discard temperature values from the heat transfer analysis. The present formulation is valid for transient heat transfer in rectangular plates, although the formulation can readily be extended to account for more complex shell geometries that have curved edges.

## 2. Element formulation

As illustrated in Fig. 2, the shell heat transfer element is discretized into $n$ layers. The thickness $t_i$ of layer $i$ is assumed to be relatively small such that the temperature field is lumped at a height $z_i$ in the layer. As a result, the temperature field in layer $i$ is reduced to a 2D temperature field that varies only in plane, as illustrated in Fig. 3. A combination of finite element and control volume techniques can be employed to arrive at a set of equations that account for heat transfer across the layers in the element. Using a technique similar to [6], the layers are assembled into a set of element equations that have the form

$$[c]\{\dot{T}\}+[k]\{T\}=\{r\} \tag{1}$$

where $[c]$=heat capacity matrix, $[k]$=conductivity matrix, $\{r\}$=an array of thermal loads, $\{T\}$=an array of nodal temperatures, and $\{\dot{T}\}$=first derivative of temperatures with respect to time.

To arrive at Eq. (1), each layer is treated as a control volume and an energy balance is performed. The equation governing 3D conduction heat transfer is generally stated as

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+Q \tag{2}$$

where $\rho$=mass density, $C$=specific heat, $k$=conductivity, $T$=temperature, $t$=time, $Q$=internal heat generation per unit volume