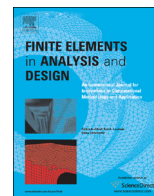




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## Multiscale finite element analysis of elastic wave scattering from localized defects

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## ABSTRACT

This paper investigates the use of a multiscale finite element approach to study the interaction between elastic waves and localized defects. The analysis of wave–defect interaction is of fundamental importance for the development of ultrasonic non-destructive testing and SHM applications. The method considered herein, known as Geometric Multiscale FEM, formulates multi-node elements which can model small geometrical features without resorting to excessive mesh refinements and without compromising the quality of the discretization in the uniform portion of the domain. The possibility of formulating libraries of damaged multiscale elements makes the method particularly appealing for conducting extensive parametric studies.

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## 1. Introduction

Among different approaches used for the monitoring of structural integrity, waves-based methods have shown great potentials for their capability to scan relatively wide portions of a structure and for being sensitive to a variety of damage types [1–4]. Typical defects encountered in structural components include cracks and corrosion in metallic materials, and porosity, delaminations, matrix and/or fiber cracking in composites. The development of effective elastic waves-based inspection techniques that not only detect damage but also provide some quantitative measure of its severity relies on numerical simulations which investigate the interaction between propagating waves and defects of different type, orientation and size [3]. Specifically, numerical simulations of wave propagation play a key role for the development of novel damage detection algorithms and are useful to support the interpretation of experimental measurements [5–7]. A concise yet meaningful understanding of these phenomena is often obtained by means of scattering diagrams illustrating the directivity of the wavefield scattered by defects [8–10]. Such simulations are computationally intensive, not only due to the scale difference between the global and local features, but also because scattering information requires several parametric studies for defects of different

size and orientation. Consequently, significant effort related to model preparation and re-meshing is often required [6].

In order to alleviate the computational cost of numerical simulations, mostly based on finite element (FE) or finite difference (FD) methods, various techniques have been developed and applied to study specific aspects of wave–damage interaction [11–13]. For example, global-local methods are based on the idea of coupling analytical solutions and FE models to compute the far-wavefield scattered by a localized defect. Applications of global-local techniques to the problem of wave scattering from cracks and inclusions in plates can be found in the early works by Koshiha et al. [14], Karim et al. [15], and Al-Nassar et al. [16]. Also, Moulin et al. [17] proposed a coupled finite element–normal modes expansion method for the analysis of Lamb waves generated by integrated transducers in composite plates. The use of analytical solutions in conjunction with a discretization approach also constitutes the basis of the Strip Element Method developed by Liu and Achenbach [18] and applied by various authors to elastic waves scattering problems [19,20]. Similar developments that employ the boundary integral equation method (BIEM) and the boundary element method (BEM) have also been proposed by Galan and Abascal [21], and Rose et al. [22–24]. These approaches allow to investigate very effectively the problem of wave scattering from arbitrary imperfections but are often limited by the scarcity of analytical expressions for complex structural/scatterer configurations.

Other methods of solution to wave–damage interactions include purely numerical approaches aimed at minimizing the number of degrees of freedom (DOFs) explicitly retained in the model.

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For example, a Bridging Scale Method (BSM) [25] is used in [26,27] to couple a coarse mesh with an overlapping fine scale mesh defined only over a limited portion of the domain near the scatterer. The onset of spurious waves at the interface between the two non-matching grids is avoided by proper bridging conditions which, however, are difficult to implement and significantly limit the performance of the method. Another approach based on non-reflecting boundary conditions has been proposed in [8,9] to characterize the scattering matrix of general defects. This approach is highly optimized for scattering calculations but unfortunately it lacks the flexibility required for general analysis purposes.

In this paper, we propose an alternative technique to model local heterogeneities based on the geometric multiscale finite element method (GMSFEM), recently developed by the authors [28]. The GMSFEM formulates multiscale elements (MSEs) which can be exploited to model heterogeneities with arbitrarily complex shapes without resorting to local mesh refinements and without inducing distortions of the FE discretization in the vicinity of irregular defects. This is achieved by providing MSEs with multiscale shape functions and elemental mass and stiffness matrices that are computed through a numerical procedure performed during the pre-processing stage of the simulation. The MSEs are then assembled along with conventional finite elements, typically quadrilaterals and hexahedral elements, as part of the global FE model. Interestingly, the GMSFEM framework can be exploited to define libraries of multiscale elements with embedded defects which can significantly reduce the effort related to model preparation and re-meshing required by parametric scattering analyses conducted with the conventional FEM.

This paper is divided into five sections including the present introduction. Section 2 provides a brief overview of the major issues related to FE modeling of irregular defects, and illustrates how the GMSFEM can be exploited to facilitate wave propagation analyses. In Section 3 an illustrative example of wave-damage interactions shows the capabilities of the proposed GMSFEM approach and validated its results. Section 4 illustrates how the GMSFEM can facilitate the calculation of the scattering coefficients of Lamb waves from a three-dimensional plate featuring different damage configurations. Finally, concluding remarks and future perspectives are summarized in Section 5.

## 2. Multiscale damage modeling approach

### 2.1. Motivation and concept

When FE or FD techniques are employed for wave propagation simulations, the size of the numerical grid is generally chosen to

spatially resolve the propagation of the waves with the shortest wavelengths [15,29], as well as the global geometric features of the object being modeled. When simulating the interaction of propagating waves with small defects, an additional constraint on the grid size is imposed by the necessity to accurately model the small scale features of the damaged area [6]. A model that satisfies all of these requirements can grow very rapidly in size as the characteristic length of the defect decreases. Consequently, the necessity to simultaneously resolve both the global and the damage scales poses outstanding numerical challenges to the current methods of analysis. Significant end-user intervention is also related to model preparation and re-meshing when conducting several parametric studies for defects of different size and orientations (Fig. 1).

In the proposed numerical scheme, the presence of localized heterogeneous regions in an otherwise uniform domain is discretized using a limited number of multiscale finite elements (MSEs). Each MSE features an arbitrary number of nodes which are used to resolve geometrical features or material discontinuities without inducing localized mesh refinements and distortions. The nodal displacements of a MSE are interpolated within the element's domain by means of an auxiliary fine scale mesh [28]. This is illustrated schematically in Fig. 2 which shows the detail of a GMSFEM discretization in the vicinity of two defects each discretized by means of a MSE. Fig. 2 also illustrates the auxiliary fine scale triangulation used within the MSEs to numerically compute their multiscale shape functions. The DOFs associated with such local mesh are denoted as *fine scale DOFs*  $\mathbf{u}$ . A subset of  $\mathbf{u}$  is retained at the coarse scale level and represents the *coarse scale DOFs*  $\mathbf{d} \in \mathbf{u}$  of the multiscale element that are assembled at the structural level. Of note is that the fine scale mesh, and the associated local DOFs, is not retained as part of the macroscopic simulation, but it is only used to compute the MSE's multiscale shape functions as described in [28]. For completeness, the method for computing the MSE's shape functions is summarized below.

### 2.2. Shape functions of multiscale elements

The interpolation functions of each MSE are obtained through a numerical solution at the local level, and are subsequently used for the formulation of the MSE's mass and stiffness matrices [28]. The vector of fine scale DOFs  $\mathbf{u}$  is partitioned in terms of the coarse scale DOFs  $\mathbf{d}$  previously defined, and a subset of local DOFs  $\mathbf{q}$  that are not explicitly retained at the macroscopic level. This relation can be generally expressed as

$$\mathbf{u} = [\mathbf{d}, \mathbf{q}]^T \in \mathbb{R}^{n_u \times 1} \quad (1)$$

where  $\mathbf{d} \in \mathbb{R}^{n_d \times 1}$  and  $\mathbf{q} \in \mathbb{R}^{n_q \times 1}$ , with  $n_d$  and  $n_q$  respectively denoting the number of coarse scale DOFs and the number of discarded

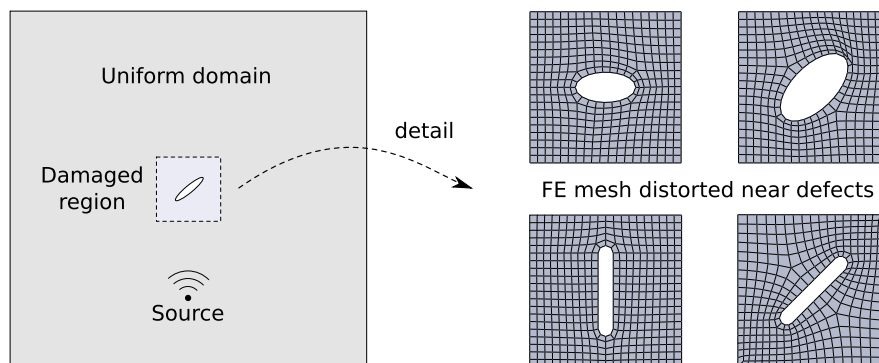


Fig. 1. Schematic illustration of a wave scattering analysis. The inserts on the left highlight the mesh distortion induced by local heterogeneities, and the need to re-mesh the entire domain when considering different types of defect.

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