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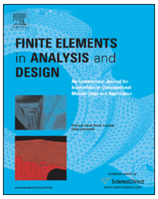

FE analysis of size effects in reinforced concrete beams without shear reinforcement based on stochastic elasto-plasticity with non-local softening

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ABSTRACT

The paper presents results of FE analysis of mechanical size effects in longitudinally reinforced concrete slender beams without shear reinforcement failing in shear mode. The simulations were performed under plane stress conditions for three beams of different sizes and a fixed shape (height/length ratio). The attention was focused on deterministic and statistical size effects related to the nominal beam shear strength. Concrete was assumed as an isotropic elasto-plastic material exhibiting non-local softening. The bond strength between concrete and reinforcement was assumed to depend on interface slip with both stable and softening responses. Statistical simulations were performed for spatially correlated Gaussian random fields of tensile strength using a stratified sampling reduction method. The FE numerical results were compared with the respective own experimental test results.

1. Introduction

The size effect phenomenon in quasi-brittle structures is related to a transition from a ductile behaviour of small specimens to a totally brittle response of large ones. Thus, the nominal strength σ_N decreases with increasing characteristic specimen dimension D. The reasons for this behaviour are: (a) intense strain localization regions with a certain volume (i.e. micro-cracked damage regions – called also fracture process zones, FPZ) which precede discrete macro-cracks; their size related to D contributes to a deterministic size effect and (b) a spatial variability/randomness of local material properties contributing to a statistical size effect that becomes dominant with increasing D.

A strong size effect also occurs in reinforced concrete beams without shear reinforcement wherein diagonal shear–tensile fracture takes place in concrete. It was experimentally observed among others by Leonhardt and Walther [1], Kani [2,3], Bhal [4], Taylor [5], Walraven [6], Chana [7], Iguro et al. [8], Bazant and Kazemi [9], Shioya et al. [10], Kim and Park [11], Grimm [12], Ghannoum [13], Kawano and Watanabe [14], Podgorniak-Stanik [15], Yoshida [16], Angelakos et al. [17], Lubell et al. [18] and Syroka-Korol [19]. The diagonal cracks at failure had in experimental tests essentially similar paths and relative lengths at the maximum load independently of the beam size. Therefore, this size effect in such reinforced concrete beams could be described by the analytical deterministic (energetic) size effect law (SEL) of Type II according to Bazant [20], being valid for structures of a positive similar geometry possessing large stress-free cracks that grow in a stable manner up to the maximum load (Fig. 1)

$$\upsilon_N(D) = \frac{\upsilon_o}{\sqrt{1+(D/D_0)}}, \tag{1}$$

Where υ_N is the nominal strength, and υ_o and D_0 are the empirical parameters depending on material properties, structure geometry and structure shape [21]. They can be determined by fitting Eq. (1) to the experimental data. The parameter D_0 separates the ductile failure ($D_0 \gg D$) from the brittle one ($D_0 \ll D$). For very large structures ($D \to \infty$), the nominal strength approaches $\upsilon_N \to D^{-1/2}$. Assuming the residual strength υ_R for very large sizes ($D \to \infty$) due to the strength of reinforcement and compressed concrete, Eq. (1) becomes valid if υ_N is replaced by the expression $\upsilon_N - \upsilon_R$ [21]. For small structures ($D \to 0$), the size effect disappears. Thus, the size effect is strong only in the limited size range. The SEL curve (Fig. 1b) in a double-logarithmic plot represents a smooth transition from a strength (plastic) limit for small sizes to the solution given by the Linear Elastic Fracture Mechanics (LEFM) for large and very large sizes.

In spite of the ample experimental evidence, the physically based size effect is not taken into account in practical design rules of engineering structures, assuring a specified safety factor with respect

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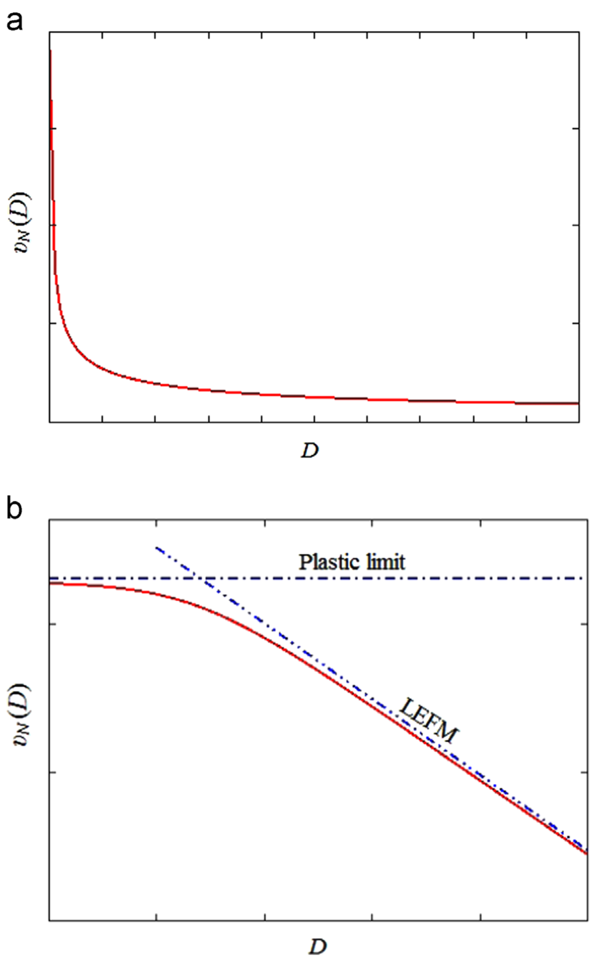

Fig. 1. Size effect curve of type II with large cracks or notches by Bazant [20,22] (v_N – nominal strength, D – characteristic specimen size, LEFM – linear elastic fracture mechanics); (a) linear scale and (b) log–log scale.

to the failure load [22,23]. Instead, a purely empirical approach is sometimes considered in building codes which is doomed to yield an incorrect formula since physical foundations are lacking.

Our objective is to provide a quantitative assessment of a size effect and a related description of a brittle failure mode in slender reinforced concrete beams without shear reinforcement under bending. The finite element method based on an isotropic elasto-plastic model with non-local softening enhanced by a characteristic length parameter of micro-structure was used in numerical studies. The plane stress 2D calculations were performed. Material parameters were calibrated with conventional laboratory tests and code recommendations. A characteristic length of microstructure was estimated by means of displacement measurements on the beam surface using the non-invasive Digital Image Correlation (DIC) technique [24]. Deterministic calculations were performed assuming a constant value of the tensile strength. In turn, statistical analyses were carried out with spatially correlated random fields according to the Gauss distribution reflecting the random nature of a local tensile strength. In order to reduce the number of FE statistical simulations, a stratified sampling scheme was used belonging to a group of variance reduced Monte Carlo methods. This approach enabled us a significant reduction of the sample number without affecting the accuracy of calculations. The present analysis constitutes the continuation of our earlier successful simulations of a combined deterministic-statistical size effect in notched [25] and unnotched concrete beams [26].

The numerical results were compared with our laboratory experiments [19,27]. The experiments were carried out on longitudinally reinforced concrete beams of different sizes and fixed height/length ratio: 3 small-size beams of the height of 200 mm and length 1500 mm, 3 medium-size beams of the height of 400 mm and length 3000 mm and 3 large-size beams of the height of 800 mm and length 6000 mm (the thickness $t=200$ mm). The beams were geometrically similar in 2 dimensions to avoid differences in the hydration heat effects which are proportional to the thickness of the member [22]. They and made from the same concrete mix with the mean aggregate diameter equal to 9 mm. The simply supported beams were subjected to 4-point bending with the constant shear span-effective height ratio equal to 3. To induce a shear–tension failure mechanism in concrete, the beams were over-reinforced without shear reinforcement (the reinforcement ratio was always $\rho=1\%$). The experimental results showed a significant size effect on the nominal shear strength versus the beam effective height. The mean nominal shear strength of large-size beams was smaller by 40% with respect to small-size beams. In all RC beams, a combined diagonal shear–tensile (significantly more tensile) and bond failure mode dominated, characterized by the development of a critical diagonal shear–tensile crack connected with a horizontal splitting crack along the top of the bottom longitudinal reinforcement toward the beam support (a shear–compression failure mechanism did not occur in concrete). The failure mode proceeded in a brittle manner in the post-critical stage.

Numerical FE analyses of slender beams without transverse reinforcement were performed among others by Kotsovos and Pavlovic [28,29], Vecchio and Swim [30], Sato et al. [31] and Slobbe et al. [32]. They used a non-linear elastic-brittle model ([28,29]), a smeared rotating crack approach [30], a smeared fixed crack approach with a sequentially linear (SL) analysis [32] and a discrete rotating crack model [31]. In calculations, a diagonal tensile brittle failure mode was usually obtained. A critical diagonal crack propagated towards the beam top, if the beam failure was caused by concrete splitting in a compression zone [32]. It propagated towards the bottom, if the beam failure was caused by bond splitting [31]. A deterministic size effect on the nominal shear strength of beams failing by diagonal tension was studied only by Sato et al. [31] for four virtual reinforced concrete beams with the height ranging from 100 mm up to 1600 mm. The numerical results were overestimated as compared to an analytical size effect formula. According to 3D simulation results in [28,29], the size effect in slender reinforced concrete beams without shear reinforcement is mainly caused by non-symmetric cracking combined with the unintended out-of-plane action, the latter being impossible to be avoided in experiments. They have also found out that stirrups eliminate the size effect in reinforced concrete beams (in contrast to recent outcomes by Yu and Bazant [33]).

Summarized, the novel elements in our calculations for reinforced concrete beams failing in shear are: (a) combined deterministic-statistical FE calculations for 3 different beams by taking strain localization and bond into account, (b) direct comparison between numerical and experimental results and (c) application of a stratified sampling scheme to reduce the number of statistical calculations. To our knowledge, such calculations have so far not been performed.

2. Constitutive models

2.1. Concrete

The concrete deformational response was simulated by assuming an isotropic elasto-plastic constitutive model with a non-local softening which was used in our previous calculations [19,26,34–36]. This relatively simple isotropic model for concrete