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A comparison of a finite element only scheme and a BEM/FEM method to compute the elastic–viscoelastic response in composite media

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ABSTRACT

This work is concerned with numerical computational procedures required to determine the mechanical response (displacements, strains and stresses) in composites built up of sectors with both elastic and viscoelastic components, in isothermal conditions. We compare a proposed scheme: a Finite Element Method Only procedure (FEM-only), with a published scheme based on an integrated Boundary Element Method/Finite Element Method (BEM/FEM) scheme. In the former, all the constituents (elastic and viscoelastic) are given a finite element discretization; whereas in the latter, the constituents are given differentiated discretizations. A novel two tier homogenization is presented: a material mathematical treatment homogenization of the viscoelastic and elastic material properties is developed and implemented resulting in a significantly efficient computational procedure; and a Dirichlet homogeneous series representation (same number of terms and same relaxation times) for all materials considered is used as well. The proposed solution structure is not viscoelastic model specific and hence significantly more general. Both model data type and/or experimentally determined data are handled with ease either separately or simultaneously. The comparison is done on the basis of applying the stated FEM-only strategy to three benchmark type problems presented in the published reference. We show that the FEM-only scheme results match rather well the actual solutions of the three problems, requiring significantly lower mathematical formulating effort and concomitantly a lower programming work. Additionally, it is shown that the FEM-only procedure is general, robust, accurate and stable, requiring relatively small computational times when implemented in personal computers. Two novel benchmark type problems are proposed for which both the analytic solution and the numerical solution are included, in order to illustrate both the generality and the accuracy of the FEM-only proposed method.

1. Introduction

The solution structure of any mechanics problem dealing with solid materials requires, fundamentally, the simultaneous consideration of equilibrium equations, strain–displacement relationships and the constitutive equations representing the mechanical behavior of the materials being used. Generally speaking, the viscoelastic component exhibiting both an instantaneous elastic response as well as a time response, where the viscous part of the behavior plays an important role, characterized by the influence of the history of mechanical events applied to the member.

Some of the typical traditional viscoelastic solutions use the analytical Laplace transform [1,2], to solve the problem in the transformed space and then, whenever possible, recover the actual solution using the corresponding inversion to the actual formulation time space. However, only the simplest of problems can be dealt with success in this fashion, pointing to the need of numerical schemes to resolve the associated analytical difficulties. In this line of work, for some applications approximate transform methods (with some form of numerical intervention) have been proposed at various stages of development in the field of viscoelasticity such as in [3,4] in the early stages, and more recently as in [5–8].

For even more complex problems, using either creep based constitutive equations or relaxation based constitutive equations, full scale numerical schemes have been used successfully in the form of Finite Element (FE) based methods [9,10], implicit time discretization schemes [11], Boundary Element (BEM) based methods [6,12–15], some of them with the limitation of being restricted to specific viscoelastic models. FE/BEM coupled procedures have been proposed and used as well [15,16]. In [16] a mixed procedure is developed and implemented to solve boundary value problems in two dimensions in the context of isothermal viscoelastic material behavior for composite domains. Given the time dependence of the problems being considered, the numerical

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schemes that have been proposed generally follow forward time marching procedures.

The aim of the present work is to compare a published BEM/FEM strategy [16] with a proposed FEM-only scheme to numerically solve problems dealing with composites made up of both elastic and viscoelastic components, in terms of their performance to produce valid numerical solutions. More specifically, we compare the results of a strategy that uses a combination of: Finite Elements to represent the elastic component, and the Boundary Element Method to represent the viscoelastic component; with results corresponding to a strategy that uses finite elements to represent the mechanical behavior of both type material components. The FEM-only procedure is applied to solve three problems: a tension bar, a sandwich plate and a reinforced tunnel, comparing the results with the corresponding ones obtained using the FE/BEM. It is shown that the FEM-only procedure solves accurately the first and second listed problems. However, for the third listed problem our results differ significantly from the published ones. Using established time limiting expressions in conjunction with formulas for thick walled elastic cylinders, we show that the FEM-only results presented herein are, in fact, the accurate ones. To resolve in a definite manner the accuracy issue we propose two novel benchmark type problems and provide both the analytic solution and the FEM-only numerical solution for each.

The FEM-only strategy, framed in the principle of virtual work, leads to a forward marching integration scheme in time. Unlike in Ref. [16], the proposed scheme works with both fundamental material behavior representations (discrete viscoelastic model defined properties) and/or experimentally determined properties as well. And also unlike the reference, it can handle, for a given problem, multiple viscoelastic component composites each with its own viscoelastic model.

Ref. [16] reports some difficulties associated with various numerical procedures when computing the instantaneous response of viscoelastic materials. As shown herein no such difficulty was encountered with the proposed scheme.

2. Constitutive and other equations of mechanics used in the computations

Although we depart somewhat from the notation used in [16] the equations to be satisfied can be shown to be equivalent. For completeness, we begin by presenting the mathematically fundamental set of equations for isothermal elastic and isothermal viscoelastic boundary value problems after [1], using standard indicial notation, as follows:

2.1. Strain–displacement equations

Employing the customary indicial notation, i and j assume the values of either 1 or 2 for the plane problems treated in this comparison, the linearized form of the strain displacements equations is used and thus strains ε_{ij} are related to displacements u_i according to:

$$2\varepsilon_{ij}(t)=u_{i,j}(t)+u_{j,i}(t) \tag{1}$$

As is usual, coma indicates differentiation with respect to the variable associated with the given index. t represents time.

The form in Eq. (1), used as well in the reference published paper [16], implies that the deformation kinematics are limited to both small deformations and small rotations only, in the composite domain.

2.2. Balance of linear and angular momentum

In the absence of body forces and volumetric moments, equilibrium by a symmetric stress tensor σ_{ij} is satisfied if the stress components are such that and comply with:

$$\sigma_{ij,j}(t)=0 \tag{2}$$

If needed, body forces and volumetric moments could readily be included in the proposed scheme.

2.3. Constitutive equations

2.3.1. Elastic material subdomains constitutive equations

The relationship between stresses and strains for isothermal, isotropic elastic members is given by

$$\sigma^e_{ij}=C^{lm}_{ij}\varepsilon^e_{lm} \tag{3}$$

where C^{lm}_{ij} represents the well known form of the linear elastic material tensor.

In a manner that will be explained in the implementation section, the alternative FEM-only proposed strategy considers the elastic components behaving viscoelastically with a fictitious dependence on time, or alternatively, viewed as a viscoelastic material that does not vary in time (infinite relaxation time). In this fashion, with either view, both elastic components and viscoelastic components are handled as viscoelastic components. The former not truly varying with time and the latter varying with time according to the chosen specific material model or material data.

In composites exhibiting both multi-elastic components and multi-viscoelastic components each elastic component is given the same representation as indicated above and treated in the manner indicated in the previous paragraph.

2.3.2. Viscoelastic material subdomains constitutive equations

The deviatoric and volumetric form of the constitutive equations for linear isothermal isotropic viscoelasticity are given, from Ref. [1], respectively by

$$S_{ij}(t)=\int_{-\infty}^{t}G_1(t-\tau)\frac{d\varepsilon_{ij}(\tau)}{d\tau}d\tau,\quad i\neq j \tag{4}$$

$$\sigma_{kk}(t)=\int_{-\infty}^{t}G_2(t-\tau)\frac{d\varepsilon_{kk}(\tau)}{d\tau}d\tau,\ \text{no summation} \tag{5}$$

where $G_1(t)$ and $G_2(t)$ are independent relaxation functions and τ represents integration time.

As usual, the deviatoric components of stress and strain are given by

$$s_{ij}=\sigma_{ij}-\tfrac{1}{3}\delta_{ij}\sigma_{kk},\quad s_{ii}=0 \tag{6}$$

$$e_{ij}=\varepsilon_{ij}-\tfrac{1}{3}\delta_{ij}\varepsilon_{kk},\quad \varepsilon_{ii}=0 \tag{7}$$

respectively, where δ_{ij} is the Kronecker delta.

We remark that expressions (4)–(7) above are given the same form in the FEM-only procedure for all possible different viscoelastic material (all models) components as will be shown.

We also remark that for the comparison being presented in this work the only viscoelastic model used is the Boltzmann solid (a particular form of the generalized Kelvin solid model) [1], (Fig. 1), given that it is the only such model used in [16]. In its one dimensional (1-D) version, is made up of two distinctive components: a 1-D spring unit in series with a 1-D viscoelastic unit, wherein a spring and a dashpot are arranged in parallel. The elastic component providing the instantaneous response, and a viscoelastic solid type component providing the delayed viscous type of response with a residual long term elastic behavior once