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An efficient response surface method and its application to structural reliability and reliability-based optimization

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ABSTRACT

In structural reliability analysis, the response surface method is widely used to reduce the computational efforts of engineering analyses. However, in order to reduce the number of finite element analysis and ensure the accuracy of evaluation, the locations of experimental points used to form a response surface function must be selected in a judicious way. Therefore, in this study, the control point of experimental points is constructed. The new center point of experimental points is chosen by using the control point instead of the design point. The control point can guarantee that the center point of experimental points lies exactly on the failure surface and is close to the actual design point. Two improved methods are proposed based on the control point and the moving technique of experimental points considering the compromise between the accuracy and the efficiency. Five examples are given to demonstrate the efficiency and the accuracy of the proposed method for both structural reliability and reliability-based structural optimization.

1. Introduction

In structural reliability analysis, the failure probability P_f is defined as [1]:

$$P_f = \int_{G(\mathbf{X})<0} f_{\mathbf{X}}(\mathbf{X})d\mathbf{X} \tag{1}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function of the vector of basic random variables $\boldsymbol{X}=[x_1,x_2,\ldots,x_n]^T$, which represents uncertain quantities such as material properties, loads, boundary conditions and geometry. $G(\mathbf{X})$ is the limit state function (LSF), $G(\mathbf{X})>0$ represents the safety domain, $G(\mathbf{X})<0$ represents the failure domain.

However, the failure probability of a given problem by means of Eq. (1) is not a straightforward task because the joint probability density function $f_{\mathbf{X}}(\mathbf{X})$ is not always available. In some cases, Eq. (1) cannot also be integrated analytically even if the $f_{\mathbf{X}}(\mathbf{X})$ is available, especially for the larger and complex structures with low failure probabilities and implicit LSFs. Therefore, in order to avoid such calculation, methods such as the first order reliability method (FORM), the second order reliability method (SORM) and Monte Carlo simulation (MCS) were proposed [2–5]. Although MCS can give the exact solution, it is time-consuming for the larger and complex structures. FORM and SORM are also difficult when the actual implicit LSF usually cannot be easily expressed explicitly. In some cases, FORM and SORM may suffer convergence problems [6]. In order to reduce the computational efforts, the response surface method (RSM) was proposed as a collection of statistical and mathematical techniques [7,8]. The basic idea of classical RSM is to approximate an implicit LSF by an equivalent polynomial function.

Several researchers proposed improvements of the classical RSM in order to evaluate efficiently the failure probability of complex structures. Bucher and Bourgund [9] proposed a quadratic polynomial response surface without cross terms. The response surface represents the LSF along the coordinate axes of the space of standard normal random variables. Rajashekhar and Ellinwood [10] proposed some ideas to improve the response surface obtained from Bucher's algorithm, in which more iterations are repeated until the convergence parameter becomes very small or zero. Kim and Na [11] proposed to arrange the experimental points in order to bring them close to the original LSF by using the gradient projection technique. Gayton [12] proposed a RSM named CQ2RS (Complete Quadratic Response Surface with ReSampling). The method takes into account the knowledge of the engineer, the statistical resampling technique is used to determine the design point. Wong et al. [13] suggested to choose a $2n+1$ axial point design and to select the parameter f as a decreasing function of the coefficient of variation of the random variables. Kaymaz and Chris [14] proposed a new response surface

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called ADAPRES (a short form of adaptive response surface method), in which a weighted regression method is applied in place of the normal regression. Duprat and Sellier [15] suggested that points efficiently positioned with respect to the design point are reused in the new iteration of the experimental design. Gavin and Yau [16] presented the use of higher order polynomials for response surface approximations. The authors proposed to use a polynomial without a fixed degree in order to fit better the LSF under investigation. Nguyen et al. [17] proposed an adaptive RSM based on a double weighted regression technique. For the first iteration, a linear response surface is chosen, for the following iterations, a quadratic response surface with cross terms is considered according to complementary points. Kang et al. [18] proposed an efficient RSM applying a moving least squares approximation instead of the traditional least squares approximation. Allaix and Carbone [19] discussed the locations of the experimental points used to evaluate parameters of the response surface. The locations of the experimental points are chosen according to the importance sensitivity of each random variable. At each iteration, the response surface is built after rotating the coordinate system. The accuracy of this method is very good in terms of the failure probability, even if a quadratic response surface with cross terms is used and the reliability index is considered as the convergence criterion.

As seen from above short literature review, most RSMs utilize an iterative approach to form the final response surface function (RSF). Thus, it is very important to find a good estimate for the design point at the first iteration for these methods since the experimental points generated in subsequent iterations are based on the result of the first iteration. If the experimental points are far from the actual design point at the first iteration, the computational efforts will be increased. In addition, the definition of a LSF is very important in reliability-based structural optimization. Since objective or constraint functions in structural analysis cannot be expressed explicitly, the sensitivity analysis must be approximately performed. It is well known that the sensitivity analysis is complicated and its computational efforts are very expensive. If the RSM is used, the sensitivity analysis will become very convenient since the partial derivatives of the functions with respect to random variables can be obtained by explicit functions. However, the classical RSM needs many experimental points to generate an approximate function.

In the paper, in order to reduce the computational efforts, the control point of experimental points is constructed. Two improved RSMs are proposed based on the control point. Since the reliability index is often used, the proposed RSM 1 focuses on the estimation of the reliability index and the design point to reduce the number of LSF evaluations. However, the method might cause some approximation error in terms of the failure probability especially for highly non-linear LSFs. Thus, in order to improve the accuracy of the estimation of the failure probability, the improved RSM 2 is proposed based on the results of the proposed RSM 1 and the moving technique of experimental points. In addition, in the proposed method, a quadratic polynomial function without cross terms is used. As already observed, this choice of the response surface enables to reduce the number of LSF evaluations. It is noted that the choice of response surface might cause some approximation error between the RSF and the actual LSF. However, if the locations of experimental points are selected in a judicious way, the approximation error in terms of the failure probability will become very small, such as the improved RSM proposed by Allaix [19].

2. Classical response surface method

In the classical RSM, the actual LSF $G(\mathbf{X})$ is replaced by a polynomial type of function $\overline{G}(\boldsymbol{X})$, typically a quadratic polynomial function without cross terms, given as

$$\overline{G}(\boldsymbol{X}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{2}$$

where n is the number of random variables $\mathbf{X}$, and a, b_i, and c_i are the $2n+1$ unknown coefficients. The unknown coefficients are obtained from discrete evaluations of the implicit LSF, such as through evaluation of the finite element method.

It is seen that Eq. (2) does not contain cross terms, hence the function $\overline{G}(\boldsymbol{X})$ basically represents the original function $G(\mathbf{X})$ along the coordinate axes. The points required to obtain $\overline{G}(\boldsymbol{X})$ are chosen to be the mean values $\overline{\boldsymbol{X}}$ and $\mathbf{X}_i = \overline{\boldsymbol{X}} \pm f\boldsymbol{\sigma}$, in which f is an arbitrary factor and σ is the vector of standard deviations of random variables $\mathbf{X}$, respectively. Next, using the $2n+1$ function values of $G(\mathbf{X})$ at these points, the parameters a, b_i, and c_i are obtained from a set of linear equations. If there are more points than $2n+1$ coefficients in Eq. (2), a least squares or similar analysis may need to be employed to best fit the surface to the points.

The original LSF cannot be properly represented by the RSF evaluated using the information obtained at the experimental points chosen in the vicinity of the mean values of basic random variables. To improve the accuracy of the RSM, Bucher and Bourgund [9] suggested an alternative process of selecting the experimental points. In the first step of this algorithm, the mean vector is selected as the center point. Then the RSF obtained is used to find an estimation of the design point $\mathbf{X}_D$. In the next step, the new center point $\mathbf{X}_M$ is chosen on a straight line from the mean vector $\overline{\boldsymbol{X}}$ to $\mathbf{X}_D$ so that $G(\mathbf{X})=0$ at the new center point $\mathbf{X}_M$ from linear interpolation, i.e.,

$$\boldsymbol{X}_M = \overline{\boldsymbol{X}} + (\boldsymbol{X}_D - \overline{\boldsymbol{X}}) \frac{G(\overline{\boldsymbol{X}})}{G(\overline{\boldsymbol{X}}) - G(\boldsymbol{X}_D)} \tag{3}$$

Next, the same interpolation is repeated to find an updated RSF$\overline{G}(\boldsymbol{X})$, as described above, by using $\mathbf{X}_M$ as the new center point.

3. Improvement of the response surface

3.1. Concept of the control point

Since the experimental points generated in subsequent iterations are based on the results of the first iteration. It is very important to find a good estimate for the design point at the first iteration. Thus, the control point of experimental points is constructed in the paper. It is explained in detail as follows.

1. Select $n+1$ initial experimental points, $\overline{\boldsymbol{X}}$ and $\mathbf{X}_i = \overline{\boldsymbol{X}} - f\boldsymbol{\sigma}$, $i=1,2,\ldots,n$, in which f is an arbitrary factor. From the case study in [20], it has been shown that very small value of f could not be used owing to numerical instability, and too large value of f would cause unrealistic experimental points. Convergence is only achieved when f is around 2 or 3 at least for the examples considered. The value of $f=3$ has been recommended by several scholars [9,14,18,19]. Thus, the value of $f=3$ is used in the paper.
2. Calculate the values of $G(\overline{\boldsymbol{X}})$ and $G(\mathbf{X}_i)$ at these points selected in step 1.
3. Calculate the differences between $G(\overline{\boldsymbol{X}})$and $G(\mathbf{X}_i)$, as follows:

$$F(\boldsymbol{X}_i) = G(\overline{\boldsymbol{X}}) - G(\boldsymbol{X}_i), \quad i = 1,2,\ldots,n. \tag{4}$$

4. The following expression is used to obtain the weight for each experiment point.

$$w_i = \frac{F(\boldsymbol{X}_i)}{\sum_{j=1}^{n} |F(\boldsymbol{X}_j)|} \quad i = 1,2,\ldots,n \tag{5}$$