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Coupling flat-top partition of unity method and finite element method

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ABSTRACT

We present a novel technique of coupling finite element method with mesh-based flat-top partition of unity method. The proposed coupling method allows us to bind any order of finite elements with flat-top partition of unity method. To verify the coupling, we test the coupling method on one- and two-dimensional boundary value problems including linear elasticity problem on a cracked domain. The coupled formulation provides a platform for stable enrichments to obtain highly accurate solution especially in the enrichment area.

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1. Introduction

In recent years, many partition of unity methods showed great success in the form of meshless method; element-free Galerkin method (EFGM) [1], reproducing Kernel particle method (RKPM) [2], method of finite sphere [3], and reproducing polynomial particle method (RPPM) [4] to name a few. On the other hand, partition of unity methods that use finite element mesh explicitly becomes popular in later years because of the easy applicability. h-p clouds [5], eXtended finite element method (XFEM) [6,7], and generalized finite element method (GFEM) [8] are in this category.

Recently proposed mesh-based construction of flat-top partition of unity function (MFPUM) [9], which uses finite element mesh explicitly, is inspired by a meshless enrichment technique [5,8,10–12]. Like GFEM or XFEM, MFPUM emphasized the use of finite element mesh to alleviate the difficulty in numerical integration. The main difference with GFEM or XFEM is the existence of flat-top in the partition of unity functions. MFPUM has the ability to enrich any order of polynomials locally with the Kronecker-delta property. Thus, imposing essential boundary conditions is straightforward as in finite element method. MFPUM is a promising method and has been successfully applied on problems that contain singularities to obtain highly accurate results. However, applying MFPUM on the entire computational domain may not be economical compared to FEM. It is more desirable to use the same order of finite elements over MFPUM on the region where the solution is smooth because MFPUM demands more DOFs compared to the same order of finite element method. Hence, it is worth to try to couple MFPUM and the finite element method.

Several techniques have been proposed to couple finite element method with different numerical methods, such as coupling finite element method with spectral method [13], with eXtended finite element method (XFEM) [14], with boundary element method (BEM) [15–17], with element free Galerkin method (EFGM) [18–21], and with moving least square method (MLS) [22]. However, the coupling method that is used in [13,15–20,22] is not applicable to couple higher order finite element method and partition of unity method.

In general, when coupling of two methods is considered, the computational domain is partitioned into two regions, and a method is chosen in each part where it is more appropriate. In such case, the continuity of the primary variable becomes an issue at the interface. Some of the commonly used technique to enforce the continuity requirement is the use of Lagrange multipliers and domain decomposition method. Both these techniques are powerful and scalable to be used for coupling of large systems. However, we prefer to have a coupling method without dealing an indefinite system and avoid iterations between subregions to get a coupled solution that is continuous. We see the coupling formulation given in [20] shares our point of view; however, the formulation, which uses mixed interpolation, is not suitable in our case.

In this paper, we will introduce a technique to couple arbitrary order of finite elements with MFPUM. To achieve a seamless coupling between these two, we develop transitional partition of unity function, which helps the transition. A schematic view of

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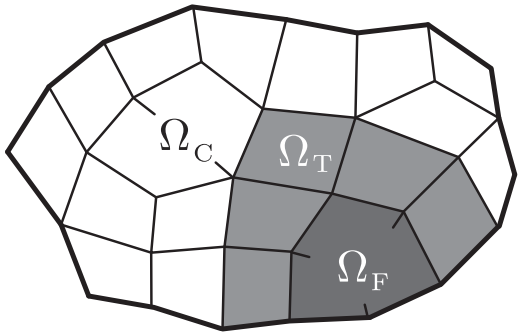

Fig. 1. Schematic partition of computational domain Ω into three parts: Ω_C where finite element is used, Ω_T where transition occurs, and Ω_F where flat-top partition of unity method is used.

such coupling in two dimensions is shown in Fig. 1. In this way, the coupled system can be dealt with a single weak form which results a positive definite system.

In the following sections, we first briefly explain mesh-based flat-top partition of unity method (MFPUM) and propose a method to couple MFPUM with FEM. To show the feasibility and effectiveness of the coupling method, we then numerically study a heat conduction problem on a bar with a boundary layer, potential flow around a cylinder and a single-edge cracked plate under uniform tension.

2. Mesh-based flat-top partition of unity method

In our previous work (MFPUM), we achieved an arbitrary order of polynomial enrichment by introducing a systematic and an efficient way of generating flat-top partition of unity functions on a *provided mesh*. Let us review the basic ideas of mesh-based flat-top partition of unity method (MFPUM) in one dimension. Let us begin by establishing some basic notations:

- *Element* E_i: A member of collection $\{E_i\}$, which partitions the domain Ω. Elements are non-overlapping, $E_i \cap E_j = \emptyset$ for $i \neq j$, and form a cover of Ω, $\bigcup_{i=1}^{N} E_i = \Omega$. In one dimension, two points ${}^{i}p_1$ and ${}^{i}p_2$ will define an element E_i. See, for example Fig. 2(a).
- *Flat-top parameter* χ_i: The flat-top size of the partition of unity function is controlled by a parameter $0 < \chi_i < 1$. Each element E_i is shrunken to E_i' by the parameter χ_i. See for example, Fig. 2(b). Each physical element E_i is defined on a local coordinate system by elemental mapping. Let T_i be the elemental mapping which maps the interval $(-1,1)$ to the element E_i. i.e. $T_i(-1) = {}^{i}p_1$ and $T_i(1) = {}^{i}p_2$. Then the mapping T_i provides the two nodes $T_i(-\chi_i) = {}^{i}p_1'$ and $T_i(\chi_i) = {}^{i}p_2'$ that defines E_i'. It is possible to control χ_i element-wise, however, we fix χ_i to be a global parameter, $\chi_i = \chi$ for $i = 1, \ldots, N$, in our numerical examples for convenience.
- *Supplemental mesh*: The interconnection between shrunken element E_i' results a supplemental mesh shown in Fig. 2(b) and will be used to build flat-top partition of unity functions. The numerical integration will be performed on the subpatches that forms supplemental mesh.
- *Patch* Q_i: A patch is a member of a covering $\{Q_i\}$, $\Omega \subset \bigcup_{i=1}^{N} Q_i$. The difference between an element E_i and a patch Q_i is that a patch can overlap with its neighboring patch(es). Fig. 2(c) shows the overlapped patches defined on the supplemental mesh. Note that the patch Q_i completely includes element E_i.
- *Partition of unity function* ϕ_i: A family of functions $\{\phi_i\}$ subordinated to each patch Q_i is called partition of unity functions if they satisfy $\sum_{i=1}^{N} \phi_i(\mathbf{x}) = 1$, $\forall \mathbf{x} \in \Omega$ where N is the total number of elements. A flat-top area of partition of unity function ϕ_i that is shown in Fig. 2(c) is given in closed form as follows:

$$\phi_i(x) = \begin{cases} \dfrac{1}{{}^{i}p' - {}^{i-1}p_2'}(x - {}^{i-1}p_2') & \text{if } x \in Q_{i-1} \backslash Q_i, \\ 1 & \text{if } x \in Q_i \backslash (Q_{i-1} \cup Q_{i+1}), \\ \dfrac{-1}{{}^{i+1}p_1' - {}^{i}p_2'}(x - {}^{i+1}p_1') & \text{if } x \in Q_{i+1} \backslash Q_i, \end{cases} \tag{1}$$

where $Q_i' = Q_i \backslash (Q_{i-1} \cup Q_{i+1})$ is the flat-top area of ϕ_i.

- *Node* ${}^{i}n_k$: The kth node of element E_i is denoted by ${}^{i}n_k$. The points $({}^{i}n_k, k = 1, \ldots, N)$ which includes the end points of the interval are distributed in the local coordinate system and then mapped to the flat-top area of the patch E_i'. Fig. 2(d) shows three nodes defined on element E_i.
- *Local approximation function* ${}^{i}L_k$: We use the Lagrange interpolating functions defined by the nodes ${}^{i}n_k$ as local approximation functions. Note that the support of Lagrange interpolating functions are unbounded, see Fig. 2(d). The local

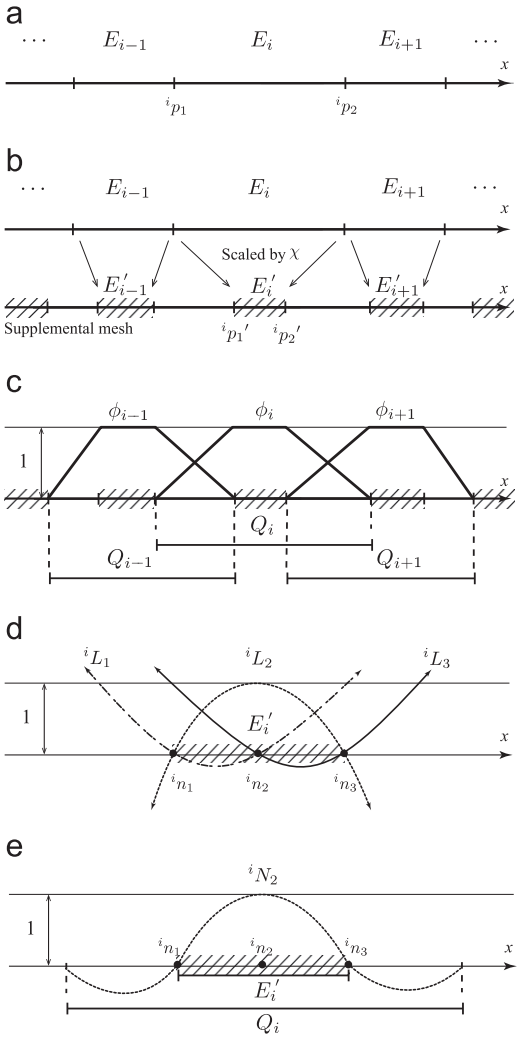

Fig. 2. MFPUM in one dimension: (a) elements and nodes; (b) shrinking elements to find flat-top regions; (c) flat-top partition of unity functions on patches; (d) second-order Lagrange interpolating functions defined by the nodes on element E_i'; (e) global quadratic approximation functions on patch Q_i.

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