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Parametric constitutive model of uni-directional fiber–matrix composite

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ABSTRACT

A computational model that allows to explicitly determine transversely isotropic elastic constants of uni-directional fiber–matrix composite tow as functions of microstructure parameters has been developed in this study. These relationships are not given in the form of analytical formulae (as it is in the case of approximate analytical models) but in the form of an extensive database of numerically evaluated results for different microstructure instances and a numerical scheme that interpolates the results. To build the database, a standard finite-element-based homogenization technique of a periodic RVE is employed. The technique is enhanced by introduction of averaging procedure over different shapes of the 2D fiber layout pattern in the tow cross-section. As a result, a numerical algorithm is provided that may be easily employed in FE codes as a part of a regular constitutive subroutine. Sensitivity of the composite elastic constants with respect to the microstructure parameters is also directly available from the model.

1. Introduction

Evaluation of constitutive properties of composite materials is a crucial issue in computational modelling of mechanical response of composite structures. The properties obviously depend on mechanical properties of the constituents as well as their geometric arrangement that implies the way they interact with each other. It has long been a challenging task to establish a closed-form relationship between e.g. the elastic constants of a composite material and parameters that define its microstructural (material and geometric) properties. Such a relationship might allow to evaluate properties of arbitrary composite with a prescribed microstructure type without the need of costly analysis or experiments. Besides, its powerful advantage would be the ability to evaluate sensitivity of material properties with respect to microstructure parameters and, in consequence, to perform parametric studies and/or numerical optimization of material microstructure.

This objective is not an easy task, though, even in the case of such geometrically simple composites as uni-directional (UD) fiber–matrix tows. UD tows are formed in wide thin tapes used as laminae in multi-layer (sandwich) laminate structures, as well as in multi-fiber yarns used to produce bi- or multi-directional woven-fabric laminae (prepregs) that again serve to build-up sandwich laminates. The UD tow is an assembly of long and thin fibers immersed in matrix made of another material. Fibers are stiffer and stronger than matrix, at least in their longitudinal direction. Matrix is considered isotropic in its mechanical properties while properties of fibers may be directional—the appropriate model for this case is deemed transverse isotropy. A variety of both fiber and matrix materials is employed in industrial practice, thus the mutual proportions between their elastic properties in composites may vary in quite wide ranges. Arrangement of fibers in the tow cross-section is random (Fig. 1a), however, the computational models typically assume some forms of regularity in their layout (Fig. 1b,c). The above listed features imply strongly anisotropic properties of the UD fiber–matrix composites: they feature high stiffness and strength in the direction of fibers and much lower in the transversal plane.

There have been several attempts made to determine the relationships between composite material constants and microstructure parameters. First, analytical methods were developed. Starting from half of the 20th century, formulae based on the theory of mixtures were employed, in which macroscopic elastic constants were resultant weighted averages of those of composite constituents, with weights related to their volume fractions, respectively. In 1960s and 1970s, more advanced geometric and material models of UD tows were formulated, among which let us mention the CCA model of Hashin and Rosen [10] and the model of Halpin an Tsai [9]. The former (Fig. 1b) leads to a mathematical formulation in which four of five transversely isotropic material constants can be exactly expressed as functions of fiber and matrix material constants and the fiber volume fraction. The latter yields formulae in which periodic model of the microstructure geometry is assumed and the resulting relationships depend on an empirical coefficient whose value reflects the

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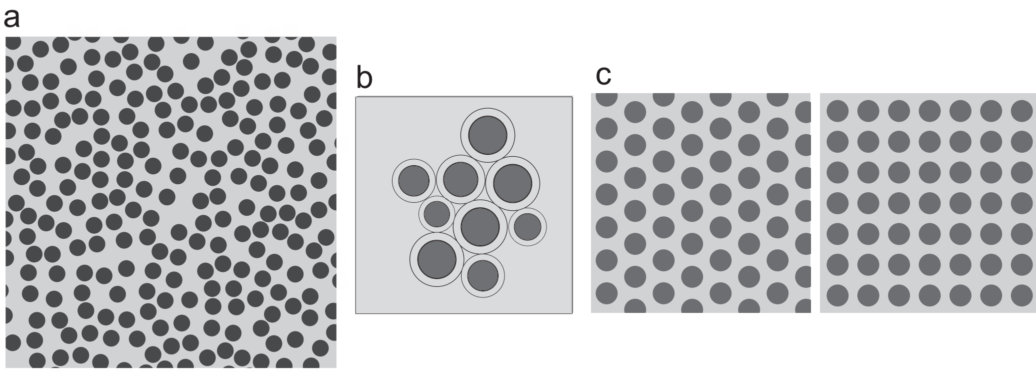

Fig. 1. UD composite cross-section: (a) actual [20], (b) concentric cylinder assemblage (CCA) model [10], (c) periodic models (hexagonal and square pattern examples).

particular geometric pattern of fiber layout. In both cases, isotropy of fibers is a necessary assumption which limits their applicability.

Significant progress has been made in this area in the end of 20th century and was due to involvement of advanced homogenization formulations [4,11]. Upon assumption of microstructure periodicity (square or hexagonal fiber layout pattern, see Fig. 1c) approximate analytical solutions, e.g. in the form of fast convergent infinite series, were proposed. A general formulation in terms of Fourier series has been given in [16], with some particular solutions (e.g. for isotropic medium with cylindrical voids). Neuman series were adapted in [3] for the case of UD tow with isotropic components in square layout. Solutions based on Weierstrass zeta functions, involving approximation of infinite coefficient matrices and vectors, have been formulated in [8,17] for square and hexagonal patterns, respectively. Although some of the solutions reach beyond assumptions of the constituents' isotropy, their generality is still limited by the predefined layout pattern. It must be noted that the results for hexagonal and square patterns are different — in the first case the homogenized material is transversely isotropic while in the other — orthotropic. The latter can be averaged over different orientations of the square pattern [2] which leads to transverse isotropy, but the differences are still present.

Subsequently, numerical methods were developed, too. Initially, assumption of periodicity and homogenization of a periodic representative volume element (RVE) were considered fundamentals of numerical formulations. Practical implementation employing FE analysis can be found e.g. in [2,19]. This assumption again enforces an arbitrary choice of a certain layout pattern of fibers—usually one of those depicted in Fig. 1c. More realistic (and much more computationally expensive) numerical models with random fiber layout were investigated in [5,6,15,20]. The resulting values of material constants were situated between those obtained for hexagonal and square periodic patterns.

The main advantage of the numerical analysis is liberation from all material and geometric simplifications that limit accuracy and completeness of analytical solutions. The full set of anisotropic material constants can be determined, also in the nonlinear, inelastic range of their behavior. Their fundamental drawback, from the point of view of our objective, is that the results are not parametrized. The evaluated material constants are only valid for the particular microstructure under consideration and provide no information about how they depend on changes of the microstructure parameters. Thus, even if some authors complete the results with their sensitivity gradients with respect to the parameters [12,13], one cannot get sufficient knowledge on how to determine elastic properties of composite at different combinations of the parameters, without multiple repeating the analysis for different data.

In this study, a parametric constitutive model of a UD composite is built for the linearly elastic range. The model consists of two parts:

- an extensive database containing values of elastic constants for a huge set of various microstructure properties, and
- an interpolation scheme for the database results.

The properties saved in the database have been determined for each case by the numerical homogenization approach with periodic fiber layout assumption and with some additional enhancement introduced in order to overcome at least some limitations implied by periodicity. As a result, a ready-to-use constitutive algorithm is presented, that allows to determine elastic constants of a UD composite tow for arbitrary values of microstructure parameters, with their sensitivity with respect to these parameters available on hand.

2. Methods

Macroscopic material properties of the composite (treated as continuum) depend on properties of its microstructure constituents (fibers and matrix) as well as of their geometric arrangement. The objective of this study is to establish the relationship between them, i.e. present the constitutive equation in a form that enables to predict the mechanical properties of a UD tow, given values of certain parameters that uniquely describe its microstructure. Consequently, one should be able to determine sensitivity of the macroscopic mechanical properties to variations of the parameters.

Let us denote by $\overline{\sigma}_{ij}$ and $\overline{\varepsilon}_{ij}$ the macroscopic (locally volume-averaged) stress and strain in the composite. If the microstructure is periodic, a repeatable RVE of the volume Ω can be defined and the averaged quantities are related to their local counterparts as

$$\overline{\sigma}_{ij} = \frac{1}{\Omega}\int_{\Omega}\sigma_{ij}\,\mathrm{d}\Omega, \quad \overline{\varepsilon}_{ij} = \frac{1}{\Omega}\int_{\Omega}\varepsilon_{ij}\,\mathrm{d}\Omega. \tag{1}$$

The local stress and strain fulfill the elastic constitutive equation

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

in which the distribution of C_{ijkl} within Ω is known. By macroscopic elastic stiffness tensor $\overline{C}_{ijkl}$ we mean the one that fulfills the following relationship:

$$\overline{\sigma}_{ij} = \overline{C}_{ijkl}\overline{\varepsilon}_{kl}. \tag{3}$$

It can be proven [19] that the averaged elastic energy density

$$\overline{U} = \frac{1}{\Omega}\int_{\Omega}\frac{1}{2}\sigma_{ij}\varepsilon_{ij}\,\mathrm{d}\Omega = \frac{1}{\Omega}\int_{\Omega}\frac{1}{2}\varepsilon_{ij}C_{ijkl}\varepsilon_{kl}\,\mathrm{d}\Omega \tag{4}$$

can be equivalently expressed as

$$\overline{U} = \tfrac{1}{2}\overline{\sigma}_{ij}\overline{\varepsilon}_{ij} = \tfrac{1}{2}\overline{\varepsilon}_{ij}\overline{C}_{ijkl}\overline{\varepsilon}_{kl}. \tag{5}$$