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A derivative-free level-set method for topology optimization ☆

David Guirguis *, Mohamed F. Aly

Department of Mechanical Engineering, American University in Cairo, New Cairo 11835, Egypt

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ABSTRACT

Topology optimization of continuum structures is a promising field that plays an important role in the design process. Although the gradient optimization methods are highly developed and succeeded in solving different problems, they are limited to problems with convex, continuous objective functions where the gradient information is known. In this paper, we propose a derivative-free level-set method using pattern search and topology description function, i.e., level-set function. The proposed approach starts with a single uniform material distribution pattern and ends by the optimized layout, without a need for prior knowledge about the objective function. In order to demonstrate the effectiveness of our approach, we tested it by solving eight benchmark problems of compliance minimization with variations in load cases, boundary conditions and topological details. The results indicate the ability of the proposed method to overcome the drawbacks of non-gradient topology optimization methods that appeared in the literature. These drawbacks include coarse finite elements (FE) meshing, checkboard pattern, inferior solutions and poor attainable topological details. In addition, the computational cost is significantly reduced.

1. Introduction

Topology optimization of continuum structures is a promising branch of structural optimization, which plays an important role in the design process. It aims at efficient allocation of the material within a predefined domain to perform one or more functions targeting maximization/minimization of one or more objectives under specified constraints. For continuum structures, the design domain is discretized into finite elements, and each element can be void or material. In mathematical presentation, the design domain is represented by a matrix of 0/1 elements that is interpreted as void/material.

As an alternative to size and shape optimization, topology optimization enables designers to obtain optimal design layouts without a previous guess. In 1988, Bendsøe and Kikuchi published a research article introducing a homogenization method for topology optimization [1]. Since this pioneering work, many numerical algorithms have been proposed which have attempted to solve problems in different fields such as structural mechanics, optics, electromagnetics and fluid. Recent reviews can be found in [2–4].

From an application's point of view, the topology optimization methods can be categorized based on the objectives and constraints into gradient and non-gradient methods. Although gradient-based methods, e.g., power-law, homogenization and conventional level-set methods, are highly developed and commonly used in a wide range of applications, they are not suitable to solve non-smooth, discontinuous, non-differentiable problems as well as problems without known gradient information. On the other hand, the non-gradient methods that rely only on the evaluation of the objective rather than the gradients or the sensitivities information may be useful in solving such problems. Examples of these problems can be found in [5–8].

Non-gradient topology optimization (NGTO) methods use stochastic search algorithms to find the optimized layouts. Such methods use simulated annealing [9], genetic algorithms [10–15], ant colonies [16], particle swarms [17] and differential evolution [18]. In addition, hybrid methods were proposed to integrate gradient and non-gradient methods for topology and shape optimization, e.g., [19–22]. Although some of these non-gradient optimization algorithms showed some success as in [15,18], the explicit formulation of the optimization problem, where each decision variable represents a single element in the design domain, is associated with some limitations such as coarse FE meshing. Other limitations of NGTO are connectivity constraint; sensitivity to initial layouts; high computational cost; obtaining sub-optimal solutions for complex problems; checkboard pattern; and search parameters that may be suitable for some problems and not for others.

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* Corresponding author. Currently at: University of Toronto, ON, Canada.
E-mail address: dsg@mie.utoronto.ca (D. Guirguis).

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As an attempt to reduce the limitation of coarse finite-element FE meshing, implicit formulations have been proposed to separate the design variables from the FE mesh. For instance, Bureerat and Limtragool [23] proposed approximate density distribution as an implicit method where the finite elements of the design domain are mapped via splines, and a filter is used to eliminate gray elements in the transformed design. Guest et al. [24] proposed an implicit formulation using Heaviside projection method [25] where the design variables are projected to the design domain with a certain length. In the work of De Ruiter and Van Keulen [26], Hamza et al. [27] and Guirguis et al. [28], implicit formulations with continuous optimization variables using a topology description function were proposed. In these formulations, the TDF in each iteration is obtained by interpolation of distributed knots within the design domain.

Although, most of these implicit methods succeeded in reducing the limitation of coarse FE meshing and checkerboard patterns, the optimization section still suffers from the other drawbacks. Our proposed NGTO method uses the formulation of topology description function that is used in [26–28], and aims to overcome some of these limitations that restrict wide use of NGTO methods in the continuum domain for real-life problems. The merits of the proposed method will be appreciated following Sigmund's criticism of the non-gradient methods [29].

The rest of this paper is organized as follows. In Section 2, the topology description function is described. Section 3 reviews the used interpolation techniques for the level-set function. Section 4 presents the pattern search method that used in the optimization phase of the proposed approach. Section 5 introduces the optimization algorithm and an analysis for the optimization parameters. In Sections 6 and 7, the carried experiments are presented and the effectiveness of the proposed technique is highlighted through a discussion of the results. This is followed by concluding remarks in Section 8.

2. Topology description function

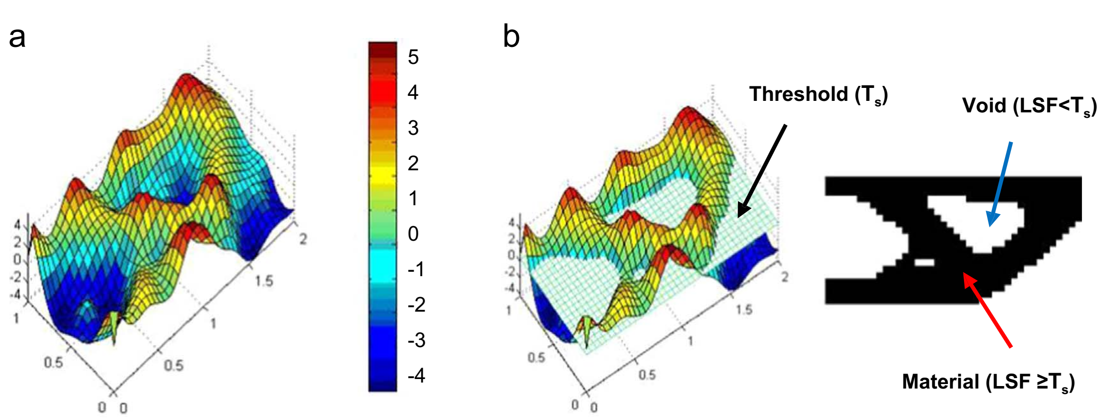
The level-set method by Osher and Sethian [30] has attracted researchers' attention and succeeded to solve problems in different fields, e.g., computer vision, image processing, fluid, combustion simulation, design and topology optimization. In conventional level-set methods, the structural contour is evolved by solving Hamilton–Jacobi PDE. In 2000, Sethian and Wiegmann [31] adopted level-set method for topology optimization, and concurrently De Ruiter and Van Keulen [32] formulated a similar level-set function based approach for topology optimization where the level-set function LSF (i.e. topology description function TDF) is evolved by a genetic algorithm (GA) rather than material interface evolution by solving PDE.

Since this start, research in the topology optimization approaches, that use level-set methods, has been in increase trend but in the direction of gradient-based methods. On the other hand, non-gradient optimization with LSF formulation often lack researchers' interest due to obtaining unsatisfactory solutions, in addition to the exhaustive computational cost that genetic algorithms require to get quasi-optimal solutions, which in turn limit GA-based methods to simple designs with few details, i.e., few design variables for limited computation resources [27].

Later on, Hamza et al. [27] used LSF formulation that was proposed by De Ruiter and Van Keulen [26,32], but using kriging interpolation of fixed knots rather than the description by superposition of radial basis functions. Guirguis et al. [33] tested the ability of interpolated level-sets to attain topological details using image-matching measures as objective functions. This study showed the capability of topology description function to attain very fine topological details; however, limitations of the optimization section were not obvious because of the smooth objective functions that were used. For randomized nature of genetic algorithms GA and its sensitivity to the initial population, multiple runs are required to ensure global solution. In addition to that, proper population size and number of generations should be increased by increasing number of variables for real-coded GA. Thus, high computational cost (i.e., hundreds of thousands of function evaluations for each run) is required to obtain designs with fine topological details.

The derivative-free level-set method for topology optimization can be defined as a topology description function that is updated or evolved by black-box optimization methods, where the solver requires only the objective value to proceed. By thresholding, the threshold plane that intersects the description function contains the layout boundaries, where the intersection curves represent the material boundaries. In uniformly meshed domains, the level-set function value above this threshold represents material, and below the threshold represents void as shown in Fig. 1. In this paper, we use the traditional representation of continuum domains, where the design domain is discretized into square cells, in order to facilitate the comparison with other approaches in the literature of NGTO. However, the intersected curves by the threshold plane can be used instead to represent the accurate, smooth boundaries of the structure by employing geometric mapping for the material-void interfaces.

There are a very large number of descriptive functions, which result in the same layout. Owing to the high degrees of freedom, this number of descriptive functions tends to be



Topology description function (Level-set function). LSF thresholding and corresponding design.

Fig. 1. Topology description function and design translation.