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On an implementation of the strain gradient plasticity with linear finite elements and reduced integration

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ABSTRACT

The size effects exhibited in the structural behaviors of micro-sized loading components cannot be described with classical plasticity theory alone. Thus, strain gradient plasticity together with appropriate experiments has been used to account for this size effect. In previous implementations of strain gradient plasticity into finite element code, low order displacement elements with reduced integration, despite their versatility for solving various structural problems, have been excluded because of their inability to yield the strain gradient inside the element. In this work, a new method of evaluating the plastic strain gradient with linear displacement elements via an isoparametric interpolation of the averaged-at-nodal plastic strain is proposed. Rate-independent yield conditions are satisfied accurately by the Taylor dislocation hardening model with Abaqus UHARD subroutine. To verify the suggested approach, the structural behaviors of micro-sized specimens subjected to bending, twisting, and nano-indentation tests were modeled and analyzed. The predicted size effects are generally in good agreement with previously published experimental results. Computational efforts are minimized and user versatilities are maximized by the proposed implementation.

1. Introduction

Classical continuum-based plasticity is limited in the context of determining the intrinsic length scale of material when the wavelength of the strain field approaches the micron or submicron size. In such cases, the stress at a point depends not only on the local strain at the corresponding point, but also on the strains in the vicinity of the point. Consequently, the stress–strain relationship becomes non-local and is described by the strain gradient. Strain gradients develop from inhomogeneous strain fields that arise due to loading patterns, presence of the crack tip, the dislocation density, grain boundaries, inclusions, and interfaces. These non-local strain fields can be identified through appropriate micro- and nano-scale experiments and observations.

Strain gradient theory was developed from the theory of linear elasticity, originally formulated by Cosserat et al. [1], who employed the work conjugate of couple stresses and the curvature tensor. Couple stresses are moments per unit area with non-symmetric Cauchy stress. The curvature tensor is a gradient of rotation. Toupin [2], Mindlin [3], and Koiter [4] later generalized the gradient theory of elongation and rotation and derived equilibrium equations with natural boundary conditions that include higher order stresses as a work conjugate of higher order strains. Constitutive relationships can be obtained by defining new strain energy density functions with higher order tensors.

Micro-scale experiments on metals during plastic deformation have revealed an evident size effect that is attributed to the plastic strain gradient. This size effect is reflected in the strengthening of the material as the specimen size is decreased to the micron or submicron level (close to the size of microstructure of the material). Such a phenomenon cannot be described by classical plasticity theory. Previous experiments on the size effect include micro-twisting tests [5], micro-indentation tests [6,7], micro-bending tests [8], and tensile tests with metal matrix composites [9,10]. From a physical point of view, the plastic strain gradient is proportional to the density of geometrically necessary dislocations.

Various theories were presented to model strain gradient plasticity. Higher-order theories, as in the Mindlin strain gradient theory, included higher-order stresses and strains in their field equations and boundary conditions (e.g., [11–14]). On the other hand, low-order theories take account of strain gradient only with the constitutive relations (e.g., [15–17]) so as to alleviate considerable complexity for the higher order strain gradient theories. Therefore, low order strain gradient theories can be implemented with finite element method in a more conventional framework of J_2 plasticity.

Crystallographic plasticity has been used to formulate dislocation density based problems [18–25]. Recently, critical assessments for

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the strain gradient plasticity theories were made either in mathematical or thermodynamic point of view [26–29].

Strain gradient plasticity in metals includes an evaluation of strain gradient with respect to various frameworks. In one evaluation, Aifantis [11,30] considered a single strain gradient invariant that is the Laplacian of equivalent plastic strain, the only length scale parameter. Fleck and Hutchinson [12] subsequently established a more elaborate theory of strain gradient plasticity with three length parameters and three invariants; they are corresponded to the rotation and stretch. However, the length parameters in this scheme must be identified through experimentation 'a priori'. Gao et al. [13] suggested a single length parameter that does not need to be experimentally determined; they related the length parameter with the Taylor dislocation model that includes a yield stress–plastic strain–plastic strain gradient relationship. Similarly, Bassani [15] proposed a single length parameter combined with single strain gradient measure of lattice incompatibility.

The majority of strain gradient problems are solved analytically and verified through experimentation (e.g., [31]). However, use of the finite element method to solve more general problems by higher order strain gradient theories is extremely difficult; the variational terms include higher order stresses and strain gradients that make utilization of the displacement method very difficult. Specifically, there are few C^1 continuous elements to satisfy the displacement, slope, and rotational continuity across the element boundary. Higher order theories also lead to difficulties in handling higher order tensors and natural boundary conditions.

Shu et al. [32] developed linear elastic mixed finite elements using a framework proposed by Fleck and Hutchinson [12]. Niordson and Hutchinson [33] developed a plane quadrilateral element having plastic strain degrees of freedom based on the similar framework. Later Wei [34] presented 3-node triangular elastoplastic element that takes full aspects of higher order theory by Fleck and Hutchinson [12]. Huang et al. [35] tested C^1-continuous element and hybrid element using a framework proposed by Gao et al. [13]. Soh and Wanji [36] developed displacement-based linear elastic triangular finite elements using the couple stress theory. The researchers also devised a patch testing scheme of constant strain and constant curvature. Fredriksson et al. [37] adopted higher order formulation of Gudmundson [27] and used plastic strain degrees of freedom for their plane quadrilateral elements. These methods were limited to the use of plane elements with increased degrees of freedom.

Niordson and Tvergaard [38] applied low order strain gradient theory proposed by Bassani [15] for their axisymmetric quadrilateral element. They calculated plastic strain gradient by interpolating effective plastic strain within the element. Qin et al. [39] and Lee and Chen [21] adopted low order formulation of Huang et al. [17] and used conventional finite elements that must have full integration scheme or higher order polynomial (quadratic or higher) element to evaluate strain gradient. Most of higher order formulations calculate strain gradient by direct differentiation of interpolated values from nodal degrees of freedom of plastic strain tensor. Alternatively, Gao and Huang [16] used a volumetric average of the integral strain in the vicinity of a material point rather than performing a direct differentiation of strain itself. In subsequent research, Abu Al-Rub and Voyiadjis [40] introduced a super element approach, while Byon and Lee [41] proposed element clustering to compute the strain gradient.

Most of prior finite element implementations based on the higher order strain gradient formulations were limited to two-dimensional with the requirement of additional degrees of freedom, state variables, and compatibility across the elements. And they tend to be considerably complex and non-standard when they are combined with a commercial package such as Abaqus [42]. However, some problems (e.g., boundary conditions in terms of plastic strain are important) can only be analyzed by the higher order theories. On the other hand, low order strain gradient formulations are relatively simple and can be implemented in the standard J_2 plasticity framework by adopting a constitutive relation incorporating strain gradient hardening. Majority of strain gradient problems were successfully solved by this scheme. However, to compute strain gradients, these formulations needed higher order polynomial elements, extra interpolation or regression of the strain field over the element cluster. Unavoidably, linear isoparameteric elements (e.g., 3 noded triangular, 4 noded quadrilateral, 4 noded tetrahedral, and 8 noded hexahedral elements) and reduced integration elements that are common practice in elastoplastic analyses were impossible or limited. Therefore, a new finite element implementation that can be applied for the reduced integration 2D and 3D linear elements is needed. In addition, modeling aspects that can handle strain gradient plasticity together with other types of failure options (e.g., various yield criteria, ductile failure, interface decohesion, and temperature dependency) are often required. These aspects, if applicable, can be integrated with Abaqus more successfully using user-defined hardening (UHARD) than user-defined material (UMAT) or user-defined element (UEL).

In this work, we propose linear finite elements for the low order strain gradient theory that can be used with reduced integration to effectively take into account the strain gradient theory in the incremental framework. The elements can be used for arbitrary mesh geometries, thus eliminating the need for super elements or element clusters within the classical plasticity framework. Since the strain gradient in the linear order elements (normally termed as constant strain element) is zero, we propose the use of averaged-at-nodal variables of plastic strain that are, in mathematical standpoint, smoothed over the surrounding elements. With this method, strain gradients and their invariants can be calculated easily as proposed. The proposed method was implemented in the Abaqus user subroutines user-defined file control (URDFIL) and UHARD, and then verified through typical micro-sized structural problems.

2. Finite element implementation

2.1. Calculation of plastic strain gradient

In the theory of strain gradient plasticity, invariants of the strain gradients are calculated in a variational framework in accordance with the corresponding length parameters. Aifantis [11,30] and other researchers (e.g., [43]) used the following strain gradient invariant of a single length parameter:

$$\overline{\eta}_A = \sqrt{\overline{\varepsilon}_{p,k}\overline{\varepsilon}_{p,k}} = \sqrt{\nabla^2 \overline{\varepsilon}_p} \tag{1}$$

where $\overline{\varepsilon}_p = \sqrt{2/3\varepsilon_{ij}^p\varepsilon_{ij}^p}$ is the equivalent plastic strain and the superscript 'p' denotes plastic variables. Fleck and Hutchinson [44] used three strain gradient invariants of three length parameters as follows:

$$\overline{E}_p^\mu = \overline{\varepsilon}_p^\mu + \ell_1^\mu\overline{\eta}_1^\mu + \ell_2^\mu\overline{\eta}_2^\mu + \ell_3^\mu\overline{\eta}_3^\mu \tag{2}$$

where $\overline{E}_p$ is the generalized equivalent plastic strain that takes into account strain gradient effects and μ is determined from the material response [45]; the three invariants were $\overline{\eta}_I = \sqrt{\eta_{ijk}^{(I)}\eta_{ijk}^{(I)}}$ and $\mu = 2$. Noting that $\rho_{ijk} = \rho_{jik} = \varepsilon_{ij,k}^p$, strain gradient components are expressed as

$$\eta_{ijk} = \eta_{jik} \equiv u_{k,ij} = \rho_{kij} + \rho_{kji} - \rho_{ijk} \tag{3}$$