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Combination of a non-local damage model for quasi-brittle materials with a mesh-adaptive finite element technique

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ABSTRACT

Progressive fracture in quasi-brittle materials is often treated via strain softening models in continuum damage mechanics. Such constitutive relations favour spurious strain localization and ill-posedness of boundary value problems. The introduction of non-local damage models together with a characteristic length parameter controlling the size of the fracture process zone is known to regularize the problem. In order to account for the non-locality of these models, it is crucial to work with fine spatial discretizations at the damage progress zone. In this paper we present a non-local damage model in combination with a mesh-adaptive finite element technique that can help automatize the analysis of progressive fracture problems in an efficient manner. Classical two-dimensional examples are given to illustrate the presented approach.

1. Introduction

The analysis of the failure of engineering materials is a subject with high interest that has been studied in the last decades from different perspectives: discrete crack models [17], extended finite element method (XFEM) [5], discrete element method (DEM) [12] or continuum damage mechanics [18]. Since this work aims at modelling the initiation of failure and not the behaviour of the material after fracturing, here the latter approach has been used because of its greater simplicity.

Continuum damage mechanics is a branch of continuum mechanics that describes the progressive loss of material integrity due to the propagation and coalescence of micro-cracks, micro-voids, and similar defects. These changes in the micro-structure lead to an irreversible material degradation, characterized by a loss of stiffness that can be observed on the macro-scale.

The term "continuum damage mechanics" was first used by Hult [18], but the concept of damage was introduced by Kachanov in 1958 in the context of creep rupture [20]. In that work Kachanov introduced the concept of effective stress, and by using continuum damage he solved problems related to creep in metals. Rabotnov [37] gave the problem physical meaning by suggesting that the reduction of the sectional area was measured by means of the damage parameter. The thermodynamic formalism involved in the irreversible process of damage was developed by Lemaitre and Chaboche [23]. Other important contributions on damage mechanics include: Mazars and Pijaudier-Cabot [27], Simo and Ju [39], Oller et al. [31], Oliver et al. [29,30], and Cervera et al. [10,9], to name but a few.

The behaviour of brittle or quasi-brittle materials such as concrete, rocks, mortar or other geo-materials is particularly difficult to predict. In those cases failure is preceded by a gradual development of a non-linear fracture process zone and a localization of strain. Realistic failure analysis of such quasi-brittle structures requires the consideration of progressive damage due to micro-cracking, modelled by a constitutive law with strain softening. This typically results in highly non-linear structural responses and so efficient non-linear solvers based on arc-length control are needed for the numerical simulations [16].

If the damage parameter depends only on the strain state at the point under consideration, and no enriched kinematics are adopted to regularize the problem, numerical simulations exhibit a pathological mesh dependence and the energy consumed by the fracture process tends to zero as the mesh is refined. This is the typical behaviour of the so-called local damage models, which are not able to properly describe both the thickness of localization and the distance between damaged zones. They suffer from mesh sensitivity (for size and alignment) and produce unreliable results. Strains concentrate in one element wide zones and the computed force-displacement curves are mesh-dependent. The reason behind these misbehaviours is that the differential equations of

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motion change their type (from elliptic to hyperbolic in static problems) and the boundary value problem becomes ill-posed [2].

Classical constitutive models require an extension in the form of a characteristic length to properly model the thickness of localized zones. Such extension can be done by means of second gradient models [11], micro-polar [28], strain gradient [40], viscous [24] and non-local terms [19]. In our model we have worked with the latter approach using a weighted spatial averaging of the internal variables. In this manner the kinematic and equilibrium equations remain standard, and the notions of stress and strain keep their usual meaning.

The first non-local models of this type, proposed in the 1960s, aimed at improving the description of elastic wave dispersions in crystals. Non-local elasticity was further developed by Eringen [15] who later extended it to non-local elasto-plasticity [14]. Subsequently, it was found that certain non-local formulations could act as efficient localization limiters with a regularizing effect on problems with strain localization [36].

Non-local models lead to smooth solutions with a continuous variation of strain. However, to resolve narrow bands of highly localized strains using the finite element method it is necessary to use sufficiently fine computational grids. Fortunately, the mesh must be fine only in the damage progression zone, while the remaining part of the structure can be reasonably well represented by a coarser mesh. In general, the localization pattern is not known in advance, and it is actually tedious to suitably construct refined meshes by hand. Thereby, the efficiency of the analysis can be greatly increased by means of an adaptive mesh refinement technique, which automates the whole process [34,6].

In the present work we present a robust non-local isotropic damage model for quasi-brittle materials that works in a small deformation regime, along with an adaptive-mesh finite element technique that permits adapting the spatial discretization in an optimal manner.

The paper is organized as follows. First, the basic concepts on continuum damage mechanics are introduced. Details are given on the basic components of the isotropic damage theory, and on the equivalent strain forms and damage evolution laws that have been implemented in this work.

Next, we review the regularization technique that has been used to overcome the problems associated to strain localization. The fundamental aspects of the integral-type non-local damage model derived are presented, pointing out the most relevant aspects of its numerical implementation. The method for estimating the error of the numerical solution and the mesh-adaptive scheme are explained in some detail.

Finally, two examples are presented showing that the combination of the non-local damage model and the mesh-adaptive technique is a robust method to model the failure of quasi-brittle materials.

2. A simple isotropic damage model

The simplest damage model for multiaxial stress states is the isotropic damage model with a simple scalar variable. This model is based on the assumption that the stiffness degradation is isotropic, i.e., the stiffness moduli corresponding to different directions decrease proportionally, independently of the direction of loading. Since an isotropic elastic material is characterized by two independent elastic constrains, a general isotropic damage model should deal with two damage variables. The model with a single variable makes use of the additional assumption that the relative reduction of all the stiffness coefficients is the same, in other words, that the Poisson's ratio is not affected by damage. The stress–strain law is postulated as

$$\boldsymbol{\sigma} = (1-d)\mathbb{E} : \boldsymbol{\varepsilon} = (1-d)\overline{\boldsymbol{\sigma}} \tag{1}$$

where $\boldsymbol{\sigma}$ is the total stress tensor, $\boldsymbol{\varepsilon}$ is the total strain tensor, $\overline{\boldsymbol{\sigma}}$ is the effective stress tensor, $\mathbb{E}$ is the elastic constitutive tensor of the intact material, and d is the scalar damage variable.

A very simple measure of the damage amplitude in a given plane is obtained by measuring the area of the intersection of all defects with that plane. Thereby, we can define the damage variable at a generic section of a material as

$$d = 1 - \frac{\overline{S}}{S} = \frac{S-\overline{S}}{S} = \frac{S_d}{S} \tag{2}$$

where S and $\overline{S}$ are respectively the total and the effective area of the section, and $S_d = S - \overline{S}$ is the damaged part of the area. An undamaged material is characterized by $d=0$. Due to propagation and coalescence of micro-defects, the damage variable grows and at the late stages of degradation process it approaches asymptotically the limit value $d=1$, corresponding to a complete damaged material with effective area reduced to zero.

In order to properly determine the evolution of the damage variable regardless of the loading case we must introduce a loading function f specifying the elastic domain and the states at which damage grows. The loading function depends on the strain tensor $\boldsymbol{\varepsilon}$, and on a variable r that controls the evolution of the elastic domain. A typical definition for function f is

$$f(\boldsymbol{\varepsilon}, r) = \varepsilon_{eq}(\boldsymbol{\varepsilon}) - r \tag{3}$$

where ε_{eq} is the equivalent strain, i.e., a scalar measure of the strain level, and r represents a scalar measure of the largest strain level ever reached in the previous deformation history of the material up to its current state, i.e.

$$r(t) = \max\left\{ r_0, \max_{\tau \le t} \varepsilon_{eq}(\tau) \right\} \tag{4}$$

The above expression implies that $r(t) \ge r_0$, where r_0 is the damage threshold, a material parameter that represents the value of equivalent strain at which damage starts. In this formula, t is not necessarily the physical time (it can be any monotonically increasing parameter controlling the loading process).

We also postulate the loading-unloading conditions in the Kuhn–Tucker form:

$$f \le 0; \quad \dot{r} \ge 0; \quad \dot{r} f = 0 \tag{5}$$

The first condition indicates that r can never be smaller than ε_{eq}, while the second one means that r cannot decrease. Finally, according to the third condition, r can grow only if the current values of ε_{eq} and r are equal.

The damage evolution law is defined as

$$d = g(r) \quad \text{with} \begin{cases} g(r) = 0 & \text{if } r = r_0 \\ 0 < g(r) \le 1 & \text{if } r > r_0 \end{cases} \tag{6}$$

which holds not only during monotonic loading but also during unloading and reloading.

There are various damage governing laws $g(r)$ that can be effectively used to model damage growth in quasi-brittle materials. In this work we adopt the exponential law proposed in [26], which separates the damage in compression and tension as

$$g(r) = \alpha_t g_t(r) + (1-\alpha_t) g_c(r) \tag{7}$$

with

$$g_t(r) = 1 - \frac{r_0(1-A_t)}{r} - A_t \exp\{-B_t(r-r_0)\} \tag{8}$$

$$g_c(r) = 1 - \frac{r_0(1-A_c)}{r} - A_c \exp\{-B_c(r-r_0)\} \tag{9}$$