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Derivation of the exact stiffness matrix of shear-deformable multi-layered beam element in partial interaction

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A B S T R A C T

This paper presents the exact finite element formulation for the analysis of partially connected shear-deformable multi-layered beams. Timoshenko's kinematic assumptions are considered for each layer or component, and the shear connection is modeled through a continuous relationship between the interface shear flow and the corresponding slip. The effect of possible transversal separation of the two adjacent layers has not been considered. The governing equations describing the behavior of a shear-deformable multi-layered beam in partial interaction consist of a set of coupled system of differential equations in which the primary variables are the slips and the shear deformations. This coupled system has been solved in closed form, and the "exact" stiffness matrix has been derived using the direct stiffness method. The latter has been implemented in a general displacement-based finite element code, and has been used to investigate the behavior of shear-deformable multi-layered beams. Both a simply supported and two continuous beams have been considered in order to assess the capability of the proposed formulation and to investigate the influence of the shear connection stiffness and span-to-depth ratios on mechanical responses of the beams. It has been found that the transverse displacement is more affected by shear flexibility than the interlayer slips.

1. Introduction

The analysis of members consisting of semi-rigidly connected layers is complicated due to the partial transfer of shear force at the interface. Over the years, there has been a great deal of research conducted on elastic two-layered composite beams in partial interaction. The first contribution is commonly attributed to Newmark et al. [1] who investigated the behavior of a two-layered beam considering that both layers are elastic and deform according to the Eurler–Bernoulli kinematics. In their paper, a closed-form solution is provided for a simply supported elastic composite beam. Since then, numerous analytical models were developed to study different aspects of the behavior of two-layered composite beams in more complicated situations. To investigate the behavior of elastic two-layered beam, several analytical formulations were proposed [2–10]. Significant development beyond that available from Newmark et al.'s paper [1] has been made in [9] by considering Timoshenko's kinematic assumptions for both layers. Besides these analytical works, several numerical models, mostly FE formulations have been developed to investigate the nonlinear behavior of both Bernoulli and Timoshenko two-layered beams with interlayer slip [11–22]. Most of the papers on layered beams in partial interaction are restricted to the case of two-layered beams and, multi-layered beams have received less attention. Chui and Barclay [23] and Schnabl et al. [24] proposed an exact analytical model for the case of three-layered members where the thickness as well as the material of the individual layers is arbitrary. Sousa et al. [25] developed an analytical solution for statically determinate multi-layered beams with the assumption that the cross-section rotation of each layer is the same although Timoshenko's kinematic is considered (cross-section rotations are the same for Bernoulli multi-layered beam). The governing equations describing the behavior of such multi-layered beam consist of a coupled system of differential equations in which the slips are considered as the primary variables. Skec et al. [26] proposed mathematical models with analytical solutions for the analysis of linear elastic Reissner multi-layered beams. The models take into account the interlayer slip and uplift of the adjacent layers, different material properties, independent transverse shear deformations, and different boundary conditions for each layer. Ranzi [27] proposed two types of displacement-based finite elements to evaluate locking problems in partial interaction of multi-layered beam based on Euler–Bernoulli kinematics. For classical polynomial shape functions, it is shown that elements without internal node suffer from the curvature locking problems. On the

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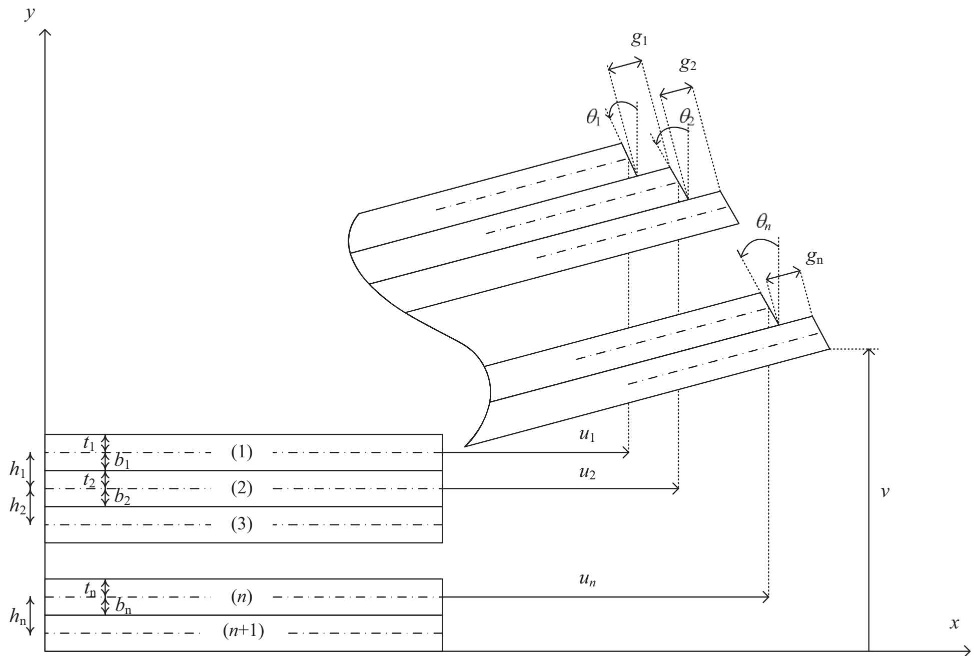

Fig. 1. Displacement field of a multi-layered beam.

contrary, adding an internal node remove locking problems, improve the representation of the axial displacement of each layer and better characterizes the partial interaction behavior of multi-layered beam.

A formulation based on the exact stiffness matrix offers the possibility of generating a locking-free model. These elements are highly attractive due to their precision, computational efficiency and mesh independency. Heinisuo [28] proposed a finite element formulation using exact stiffness matrix for uniform, straight, linearly elastic beams with two faces and one core and with three symmetric faces and two identical cores. Based on the analytical solution given in [25], Sousa [29] derived the exact flexibility matrix for partially connected multi-layered beams with the assumption that both transverse displacement and rotation are the same for all layers. The model is based on the derivation of flexibility matrix obtained from a statically determinate coordinate system.

The purpose of this paper is to present a new exact FE formulation for the analysis of shear-deformable multi-layered beams in partial interaction based on the exact stiffness matrix derived from the governing equations of the problem. The features of the formulation presented in this paper are as follows: (i) longitudinal partial interaction of the layers is considered which provides a general description of the stresses and strains in the layers; (ii) different rotations of cross-sections of each layer are considered; (iii) exact stiffness matrix is used which provide accurate and stable results. The present model provides, therefore, an efficient tool for linear elastic analysis of shear-deformable multi-layered beam with arbitrary support and loading conditions. The rest of the paper is organized as follows. In Section 2, the field equations for a shear-deformable multi-layered beam in partial interaction are presented. The governing equations of the problem are derived in Section 3. In Section 4, the full analytical solution of the coupled differential governing equations is provided, regardless of the loading and the nature of the boundary conditions (support and end force). Special care has been taken while dealing with the constants of integration. The exact expression for the stiffness matrix is deduced for a generic shear-deformable multi-layered beam element in Section 5. Numerical examples are presented in Section 6 in order to assess the performance of the formulation and to support the conclusions drawn in Section 7.

2. Field equations

The field equations describing the geometrically linear behavior of an elastic multi-layered beam with $n+1$ layers in partial interaction are briefly outlined in this section. The following assumptions are commonly accepted in the model to be discussed in this paper:

- connected members are made out of elastic, homogenous and isotropic materials;
- the cross-sections of all components remain plane but not orthogonal to beam axis after deformation;
- adjacent layers are connected at their interfaces where relative slips can develop;
- transverse displacement v is assumed to be the same for all layers;
- discretely located shear connectors are regarded as continuous.

Quantities with subscript sc are associated with the interface connection.

2.1. Compatibility

With the above assumptions, the layer has independent cross-section rotation and curvature, see Fig. 1. Kinematic equations relating the displacement components (u_i, v, θ_i) to the corresponding strain components $(\epsilon_i, \theta_i, \kappa_i)$ are as follows:

$$\epsilon_i = \partial u_i \tag{1}$$

$$\theta_i = \partial v - \gamma_i \tag{2}$$

$$\kappa_i = \partial \theta_i \tag{3}$$

where

- $\partial \bullet = \mathrm{d}\bullet/\mathrm{d}x$;