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# Static analysis of tapered FRP transmission poles using finite element method

B. Saboori [a,*], S.M.R. Khalili [a,b]

[a] Centre of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Pardis St., Molasadra Ave., Vanak Sq., Tehran, Iran
[b] Faculty of Engineering, Kingston University, London, UK

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ABSTRACT

This paper deals with linear static analysis of tapered FRP (fibre-reinforced polymers) transmission poles with circular thin-walled cross-section using a second order shell element and first order shear deformation theory (FSDT). The cross-section is treated as a generally orthotropic laminate. Typical poles are analyzed using a provided computer code by MATHEMATICA program. The results obtained by presented numerical modeling are verified through comparing to analytical results or those obtained from ANSYS 11.0 commercial finite element program. The effects of various parameters such as fibre orientation and type, fibre volume fraction, number of layers and geometry on the behavior of tapered FRP transmission poles are investigated.



## 1. Introduction

The unique characteristics of fibre-reinforced polymers (FRP) such as high strength-to-weight ratio, resistance to corrosion, and lower transportation, installation and maintenance costs, have made them suitable to use for manufacturing power transmission poles as a replacement of conventional materials (wood, concrete and steel). Transmission poles are mostly tapered and thin-walled members and are mainly subjected to cantilever bending due to wind gusts and cables tension. Moreover, because of the weight of cross arms and other equipments installed at the top, axial load is also applied to the poles. The poles are made of polymer matrix with reinforcing fibres, fabricated by filament winding technique. Polyester, vinylester or epoxy is mainly used as matrix and E-glass, S-glass, aramid or carbon fibres are as reinforcement [1].

The theoretical investigations on fibre-reinforced polymer poles are very limited. To analyze FRP poles, the theory of shell of revolution may be used, since tapered FRP poles can be considered as rotational shells [2]. Due to the complexity of material properties, layer lay-ups, fibre orientations, tapered shape, and combination of loads on FRP transmission poles, the pure analytical method is incapable of providing acceptable solutions effectively. On the other hand, the numerical analysis methods are proved to be more efficient. Since finite element method is one of the most efficient methods of numerical analysis in structural engineering, its combination with multilayered shell theory can be used to calculate the state of deformation and stress of the FRP poles.

Navaratna et al. [3] were among the early researchers to use the finite element method to analyze the stability of shells of revolution. The approach used by them to develop the geometric stiffness matrix was followed by Gould and Basu [4] for linear buckling analysis and incremental deformation analysis of rotational shells. Ugural and Cheng [5] investigated the linear buckling of composite cylindrical shells under pure bending. Holston [6] investigated the buckling of filament-wound cylinder under axial compression. Gould [7] dedicated the subject of finite element analysis of shells of revolution in his book. Fourier series are employed to represent displacement functions of rotational shells. Noor et al. [8] assessed a group of computational methods for multilayered composite cylinders. The finite element models used are mostly two or three dimensional elements. Khdeir et al. [9] studied the vibration and buckling of cross-ply circular cylindrical shells with various shell theories. The different approaches of inclusion of transverse shear strains are discussed. Caracoglia and Jones [10] performed a numerical and experimental study of vibration mitigation for highway tapered aluminum poles. The study is performed in order to identify the potential causes associated with the failures. It is concluded that, although the poles are designed according to standard specifications, an unusual event in which the combination of wind and frozen precipitation was observed, could be responsible for large vibration amplitudes. Polyzois et al. [1] investigated the free vibration of tapered glass fibre reinforced polymer (GFRP) poles using a tapered beam element. In their work, the natural frequency and period of the fundamental flexural vibration mode of the pole are computed for various cases of lamination angle, taper ratio, and mass at the top. Raftoyiannis and Polyzois [11] surveyed the effect of semi-rigid connections on the free vibration of tapered GFRP poles by finite element method (FEM) and found that the joint

* Corresponding author. Tel.: +98 2184063208; fax: +98 2188674748.
*E-mail address:* saboori.b@gmail.com (B. Saboori).

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flexibility mainly affects the vibration behavior of the pole in the cases of weak connections, while for sufficiently strong connections, this effect becomes negligible. FEM was used by Ibrahim and Polyzois [12] to analyze the cross-section ovalization behavior of FRP poles under a bending load. Based on their results, circumferential layers tend to increase the critical ovalization load of FRP poles. Ibrahim et al. [13] also tested a number of full-scaled GFRP poles under cantilever bending up to failure and utilized FEM to evaluate failure load. The performance characteristics of tested poles suggest that a lower safety factor than the values in the standards can be used for the design of FRP poles. In the work of Caracoglia [14] two problems are analyzed: the susceptibility to across-wind galloping-type vibration associated with the deposit of frozen-precipitation on the surface of the poles or the luminaire installed at the top of the unit; and the influence of eccentric aerodynamic loading at the level of the luminaire on the dry-unit buffeting response. Caracoglia and Velazquez [15] also compared the dynamic performance of steel, aluminum and GFRP light poles through experimental testing. The comparison of the performances is based on frequency and damping ratios corresponding to the first and second-mode vibrations. Transient dynamic analysis of tapered FRP transmission poles was recently investigated using tapered beam finite element and precise time integration method by Khalili and Saboori [16].

For large displacements, the load-deflection curve obtained through the linear analysis of the FRP poles deviates significantly from the nonlinear curve. But in small displacements (pole tip deflection/pole length $<0.25$), the linear solution is a good approximation to the nonlinear solution [2]. Because the linear analysis is much easier to manipulate than the nonlinear analysis and the linear program is less time-consuming and efficient, most of the parameters incorporated in the analysis, such as taper ratio and fibre orientation, can be determined by performing a linear analysis in the design of FRP poles. Such analysis could give reasonable results if the pole tip deflection/pole length ratio is $<0.25$.

The present study, deals with the linear static analysis of the FRP transmission poles and evaluating the various effective parameters on their behavior using a second-order shell finite element model. In this analysis, the principle of stationary potential energy is employed to establish equilibrium and stability conditions. To account for not only the beam-column-type, but also the shell-type behavior, the theory of shell of revolution is utilized and the strain energy is formulated. Based on the performed formulation, a computer code is provided by MATHEMATICA software for static analysis in small displacements. Once the results obtained from the finite element model for a typical FRP pole were verified through the comparison with ANSYS commercial software and analytical results, the effect of various parameters such as geometric characteristics, fibre's type and orientation and fibre volume fraction on the static behavior of tapered FRP poles are investigated.

## 2. Basic constitutive equations for FRP

The cross-section of FRP poles with the general geometry shown in Fig. 1, is made of a combination of several laminas. The fibres in each lamina are unidirectional, but with different orientations in various laminas. A single layer of composite can be treated as an orthotropic lamina with respect to its principal material coordinates 1–2–3 (1-axis is in the fibre direction). The stress–strain relationship is [17]

$$\begin{Bmatrix} \sigma_1^m \\ \sigma_2^m \\ \tau_{23}^m \\ \tau_{31}^m \\ \tau_{12}^m \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^m \\ \varepsilon_2^m \\ \gamma_{23}^m \\ \gamma_{31}^m \\ \gamma_{12}^m \end{Bmatrix} \tag{1}$$

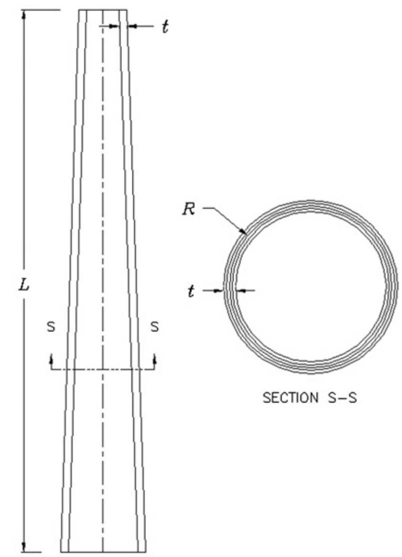


**Fig. 1.** General geometry of a tapered composite pole [1].

or

$$\{\sigma^m\} = C\{\varepsilon^m\} \tag{2}$$

where $C$ is the material stiffness matrix. Superscript "$m$" denotes that the stresses or strains are expressed in material coordinates. Since wall thickness of the pole is very small with respect to the other dimensions, the stress along the thickness is negligible and the generalized plane stress assumption [18] will be true ($\sigma_3^m \approx 0$).

The six material constants can be expressed in terms of engineering material constants [17]:

$$C_{11} = \frac{E_L}{1-\upsilon_{LT}\upsilon_{TL}} \tag{3}$$

$$C_{22} = \frac{E_T}{1-\upsilon_{LT}\upsilon_{TL}} \tag{4}$$

$$C_{12} = \frac{\upsilon_{LT}E_T}{1-\upsilon_{LT}\upsilon_{TL}} = \frac{\upsilon_{TL}E_L}{1-\upsilon_{LT}\upsilon_{TL}} \tag{5}$$

$$C_{44} = G_{TT'} = \frac{E_T}{2(1+\upsilon_{TT'})} \tag{6}$$

$$C_{55} = G_{LT'} = G_{LT} \tag{7}$$

$$C_{66} = G_{LT} \tag{8}$$

where $L$ denotes the fibre longitudinal direction ($L=1$), $T$ denotes the fibre transverse direction ($T=2$) and $T'$ ($T'=3$) denotes the normal (to the lamina) direction. Only five engineering material constants are needed to determine these six material constants. They are: $E_L$, the longitudinal elastic modulus; $E_T$, the transverse elastic modulus; $\upsilon_{LT}$, the major Poisson's ratio; $\upsilon_{TT'}$, Poisson's ratio in transverse directions; and $G_{LT}$, the shear modulus (longitudinal and transverse directions). The minor Poisson's ratio $\upsilon_{TL}$ can be determined by $\upsilon_{TL}=\upsilon_{LT}\,(E_T/E_L)$ [17].

In the following analyses, stresses and strains are referred to structural global coordinates ($x$–$y$–$z$). For a typical lamina, the relationship of the material (local) coordinates and the structural coordinates is that the $z$-axis coincides with the 3-axis, while the 1-axis is rotated an angle $\theta$ with respect to the $x$-axis. Accordingly, the stress–strain relationship is found as

$$\{\sigma\} = \overline{C}\{\varepsilon^*\} \tag{9}$$