Finite Elements in Analysis and Design 107 (2015) 1–12

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Modeling contact of pipes within elliptic and spiralled wellbores using finite element analysis

H.M. Panayirci

Schlumberger Gould Research Center, High Cross, Madingley Road, Cambridge CB3 0EL, UK

ARTICLE INFO

Article history:
Received 26 February 2015
Received in revised form
1 July 2015
Accepted 10 August 2015
Available online 2 September 2015

Keywords:
Nonlinear finite element analysis
Contact modeling
Drilling applications

ABSTRACT

This paper addresses the formulation and implementation of a contact algorithm for circular pipes within elliptic and spiralled wellbores using nonlinear finite element analysis. The need for such a capability originates from drilling applications in oil industry. Normally, the drilling bits have circular cross sections and the intention is to drill perfectly circular holes. However, it is quite common that the drilled wellbores show irregularities. This study derives the formulations to calculate the clearances between the pipes and such boreholes, which are essential to detect contact and calculate the associated contact forces. The formulations are described in detail and the implementations and flowcharts are provided as well. A case study is presented to reveal the implications to have such capabilities within a finite element engine, such as capturing the damage on the drillpipes in terms of bending moments.

1. Introduction

The finite element (FE) method is probably the most widely used tool for structural analysis of various engineering applications. In this regard, drilling and completion operations within oil industry are no exception. For these cases, one could be interested in calculating the static equilibrium of drillpipes in wellbores. This would reveal the equilibrium configuration of the pipe and all the forces acting on it. To achieve this, these models must handle the so-called "contact problem", since we are dealing with pipes in very tight boreholes. This implies calculating the maximum allowed movement of the pipe within its confined space. One should note that an accurate description of the wellbore would be the core component in that sense. If available, such a description would allow us to determine the effect of the hole more realistically and estimate the success of the planned operations, such as running pipe, cementing, logging, and performing completions. We should keep in mind that no matter how complex or advanced the model, the output is heavily dependent on the accuracy and detail of the modelled wellbore, which eventually determines the mechanical condition of the pipe inside the hole. Therefore, it is crucial to have the capability of capturing the effect of irregular hole shapes in our model, since we might not always end up with perfectly smooth circular holes.

This work presents the formulation and implementation to enable modeling drillpipes within irregular (noncircular or oscillatory) borehole shapes using an FE engine. First, a brief background on the basics of the FE model will be provided. Then, the problem at hand will be described mathematically for which analytical solutions will be provided. The details on the implementation will be also reported. Finally, the model will be used to understand the effect of wellbore irregularities on the positioning and bending moments of the pipe.

2. Theoretical background

Various models are used in oil industry to estimate the forces on drillpipes [1–6]. These tools provide various benefits, such as estimating torsional and axial drag, bending moments, load distribution along the drillpipe and critical buckling loads for a given hole-pipe-wellbore combination. Such a model has been developed in Schlumberger Gould Research center, where 3D beam elements are used within a static nonlinear FE formulation. It is worth mentioning that the model employs the so-called Co-rotational formulation [7], where the motion of every element is decomposed into rigid body motion and pure deformations through the use of an additional reference system. This reference system is called CR reference, which continuously rotates and translates with the element. By doing so, the small deformations are captured in local (element) coordinate system, while the geometric nonlinearities are handled by extracting the rigid body motion from the overall displacements. Hence, this approach has been recognized to be especially suitable for structures with large

E-mail address: hpanayirci@slb.com

http://dx.doi.org/10.1016/j.finel.2015.08.002
0168-874X/

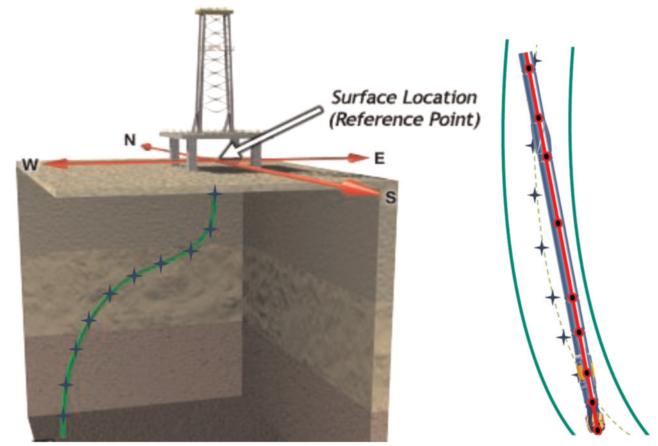

Fig. 1. Left: 3D view of a wellbore. Right: discretized drillpipe within the wellbore trajectory.

displacements and rotations, but with small strains. Considering the drillpipe, one can note that it will be often subjected to large displacements and rotations in order to fit within the drilled wellbore, while the deformations at local element level will be low. This enables the use of high performing local element formulations. For the sake of completeness, we will describe some basic aspects of the nonlinear FE model used in the following (see Appendix B for the details on the Co-rotational FE formulation and [8,9] for details on nonlinear FE analysis).

The FE analysis of a drillpipe starts with the description of the wellbore profile as shown in Fig. 1 on the left. Usually, one is provided with the description of the trajectory, which are obtained from measurements during and/or after drilling. This information is then used the determine coordinates of well points (shown with dark blue stars), which represent the drilled hole in a discretized manner. It is important to note that these well points construct the reference coordinate system for the FE analysis, i.e., the displacements are solved with respect wellbore center, using the following equilibrium condition,

$$\mathbf{f}_{\mathbf{int}} = \mathbf{K}\mathbf{u} = \mathbf{f}_{\mathbf{ext}} \tag{1}$$

where $\mathbf{K}$ is the $n \times n$ stiffness matrix (n being the number of degrees of freedom), $\mathbf{f}_{\mathbf{int}}$ and $\mathbf{f}_{\mathbf{ext}}$ are the $n \times 1$ internal and external force vectors, respectively, and finally $\mathbf{u}$ is the $n \times 1$ displacement vector to be solved. Here, the stiffness matrix is assembled for the deformed shape of the structure and hence is a function of displacements,

$$\mathbf{K}(\mathbf{u}) = \frac{\partial \mathbf{f}_{\mathbf{int}}}{\partial \mathbf{u}} \tag{2}$$

Consequently, the solution of the nonlinear FE analysis is performed iteratively using a Newton–Raphson algorithm, where the stiffness and the internal forces are re-calculated for every change in the displacement vector. The iterations are continued until the equilibrium between the internal and external forces of the structure is achieved or the incremental change in the displacements are negligible. This procedure can be summarized as follows:

1. Assemble the external force vector $\mathbf{f}_{\mathbf{ext}}$ considering the weight of the pipe and applied loads.
2. Calculate $\mathbf{K}^{\mathbf{e}}(\mathbf{u})$ and $\mathbf{f}^{\mathbf{e}}_{\mathbf{int}}(\mathbf{u})$ for every element and assemble the stiffness matrix and internal force vector for the whole structure.
3. Apply boundary conditions.
4. Check contact for every node and apply contact force if necessary.
5. Solve for incremental displacements $\Delta\mathbf{u}^{\mathbf{i}}$
$$\mathbf{K}(\mathbf{u}^{\mathbf{i}})\Delta\mathbf{u}^{\mathbf{i}} = \mathbf{f}_{\mathbf{int}}(\mathbf{u}^{\mathbf{i}}) - \mathbf{f}_{\mathbf{ext}} \tag{3}$$
6. Update the total displacement vector as
$$\mathbf{u}^{\mathbf{i}+\mathbf{1}} = \mathbf{u}^{\mathbf{i}} + \Delta\mathbf{u}^{\mathbf{i}} \tag{4}$$
7. Check convergence by comparing the norms of the incremental displacements $\|\Delta\mathbf{u}^{\mathbf{i}}\|$ and residual force $\|\mathbf{f}_{\mathbf{res}} = \mathbf{f}_{\mathbf{int}} - \mathbf{f}_{\mathbf{ext}}\|$ to pre-set threshold values
8. Repeat steps 2–7 until convergence is achieved

The procedure described above is a common implementation for nonlinear static FE analysis and hence will not be elaborated any further. However, for the sake of clarity, we will spare a few sentences here regarding the application of contact. It is worth noting that while creating the wellbore input for the FE engine, an individual diameter is assigned for every well point used for the discretization of the trajectory. In addition, the mesh for the drillpipe will contain naturally the diameter for every beam element. These two sets of data are then used together with another algorithm, which identifies the closest wellpoint to every node according to the depth of the drillpipe. All these allow the FE engine to determine and apply contact (step 4 in the above algorithm), which will be described in the next section.

3. Problem definition

From a geometrical point of view, there are three main components required for a complete wellbore description: the location of hole centerline, hole size, and hole shape. Such an input to any related model would improve the quality of its output substantially. This is, of course, provided that the FE model can process the detailed wellbore input. In this regard, one critical feature missing in current FE models used in oil industry is taking the variable clearance (due to the noncircular hole shape) into account.

The reason for this is simply the additional complication introduced into the contact algorithm as explained in the following. To start with, let us define some basic concepts for the simple case. Within contact mechanics, clearance is defined as the maximum amount of distance over which a structure is allowed to move in a certain direction before contact occurs. Considering our problem, in which the drillpipe is represented using beam elements with circular cross sections, the calculation of clearance poses no difficulty for circular wellbores. That is, the clearance (marked with symbol c in Fig. 2) for any node can be easily deducted from the dimensions of the pipe and wellbore at the considered location. This clearance will be a single value, independent of the direction of the pipe movement. As a result, the contact can be easily detected by comparing the total lateral displacement of the pipe (u) against the clearance and the resulting contact force can be calculated using a penalty formulation (see Appendix C for the details), which is a commonly used approach among many other contact force formulations available in the literature [10]. Furthermore, the direction of the contact force introduces here no additional issues, since it will be simply aligned with the displacement direction of the pipe (as shown in Fig. 2).

One can immediately see however that this assumption falls apart, if the wellbore is not circular. For noncircular hole shapes, one needs to calculate the clearance depending on the pipe movement, as the pipe can move different distances towards different directions before contacting the wellbore. In the following