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Elastodynamic infinite elements with united shape functions for soil–structure interaction

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ABSTRACT

In this paper, new formulations of 2D four and five node *elastodynamic infinite elements with united shape functions* (*EIEUSF*) are proposed. Such elements can be treated as family elastodynamic infinite elements, appropriate for dynamic soil–structure interaction (SSI) problems and can be used directly as finite elements. The common characteristic of the proposed infinite elements is the so-called united shape function, which is based on a finite number of wave shape functions. The idea of the united shape function is presented and discussed in details. It is demonstrated that if only one wave function is used (only one wave frequency) these elements are transformed to single-wave elastodynamic infinite elements.

The basic difference between the four and five node formulations is the number of nodes for interpolation of the displacement field in the finite direction of the element. The *EIEUSF* elements are given in decay and mapped form. Fundamental advantage of the proposed infinite elements is that such elements can directly be used in the FEM concept. Once generated the *EIEUSF* mass and stiffness matrices can be addressed, introduced appropriately in the FEM equation of motion. After the mapping such elements can be recognized and treated, in the FEM technology, as finite elements.

The elements are tested for multi-wave soil–structure interaction problems. The results give the guarantee for adequate numerical simulation.

1. Introduction

Exterior domain-scattering problems appear naturally in many engineering fields such as electrodynamics, magnetic problems, fluid flow, thermal analyses, etc. Wave propagation in an elastic infinite media and the scattering of waves on bodies in a fluid which extends infinitely are of particular interest. The main difficulty in such problems when we use numerical methods arises in an unbounded domain that has to be discretized. Many suggestions and ideas for the treatment of the exterior domain have been presented and discussed in a number of research papers for the last three decades. An exterior (infinite) domain cannot be completely discretized with standard finite elements, so a lot of effort has been spent in the development of new infinite element techniques.

One possible approach is to just truncate the computational domain at some distance away and to impose "appropriate" boundary conditions. Such a boundary is called an "artificial" boundary. In this case the so-called viscous, absorbing or transmitting boundary conditions can also be used. It is evident that the computational efficiency depends more on the localization of the "artificial" boundary and the type of boundary conditions. In a lot of problems such techniques provide acceptable results. In soil–structure interaction problems such techniques are known as a *substructural approach*.

The basic idea in this research is to couple the advantages of the finite element method with some infinite element approximation techniques in order to simulate wave propagation in elastic media more properly. This simulation is very important in a broad class of problems in computational mechanics. In structural mechanics, for instance, these are soil–structure interaction problems. For adequate results in soil–structure interaction problems it is very important to consider energy radiation due to the vibration of the structure into the far field medium. SSI effects very often play an essential role, especially in the case of stiff and massive structures resting on relatively soft ground. In static analysis, the simple truncation of the far field with the setting of appropriate boundary conditions often gives very good results. However, in dynamic cases, an artificial boundary made by truncation causes the results to be erroneous because of the reflection of the waves. In recent decades, much work has been done on unbounded domain problems, and several kinds of modeling techniques have been developed to avoid these effects. Such techniques are viscous boundary, transmitting boundary, boundary elements, infinite elements and the system

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identification method. At the same time, several numerical methods for these types of problems have been suggested. The basic idea of these approaches is to divide domain Ω into two parts, the bounded part Ω_c and unbounded part Ω_∞, where for the first one $x_i \le c_i$ is valid. For more appropriate simulations we need to set the assumption $u(x_i)=0$ on Ω_∞.

Among these approaches, using infinite elements is good way to solve soil–structure interaction problems, since their concept and formulation are much closer to those of the finite element method except for the infinite extent of the element domain and the shape function in one direction. In this case, there is no loss of symmetry of the element matrices. The unbounded domain Ω_∞ is partitioned into a finite number of infinite elements directly incorporated with the element mesh in the bounded domain Ω_c. In the numerical models these domains very often are called near (Ω_c) and far (Ω_∞) fields.

A very important advantage when mapped infinite elements are used in the finite element method concept is the validity of the *p* (polynomial degree) version, *h* (refinement) version and *ph* (adaptive) version of this method. So the accuracy of the numerical solution depends only on the finite element method. From the algorithmic point of view, the infinite element is treated as a standard finite element except for the infinite approximation in the infinite direction.

In short, the structure (situated in the near field) and the near field are discretized with the finite elements. The part outside the truncation (artificial boundary) called the far field is discretized by the infinite elements discussed below. The continuity across the finite and infinite elements is enforced in exactly the same way as between two finite elements.

2. Backgrounds of infinite elements

Infinite elements were originally introduced by Ungeless and Bettess [1] about three decades ago. Now this technique is one of the most often used since its concept, and the formulation is much closer to those of the finite element method. These elements are very effective for the modeling of structures with a near field presented by finite elements and a far field presented by infinite elements. In recent years a lot of dynamic infinite elements have been developed.

The local coordinate system of one proposed decay infinite element is shown in Fig. 1. The geometric mapping (in the far field) from the local coordinates to the global coordinates for the *HIE* kind of infinite element is defined as

$$x = x_b(1+\xi) \quad \text{and} \quad z = \sum_{i=1}^{n} L_i(\eta) z_i,$$

where x_b is the global coordinate in the x direction of the artificial boundary between the near field and the far field, (Fig. 2), n is the number of nodes for the infinite elements, and $L_i(\eta)$ is a Lagrange polynomial, which has a unit value at the ith node while zeros are at the other nodes. For this element the ranges of the local coordinates are: $\eta \in [-1,1]$ and $\xi \in [0,\infty)$.

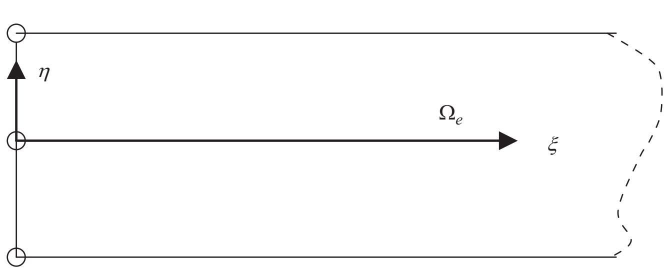

Fig. 1. Local coordinate system of horizontal infinite elements (HIE), $\xi \to \infty$.

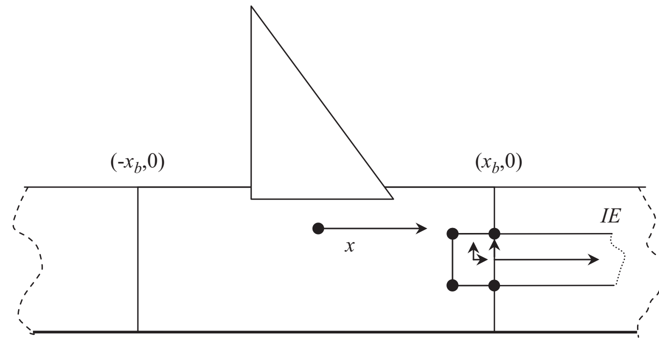

Fig. 2. Artificial boundary between the near field and the far field.

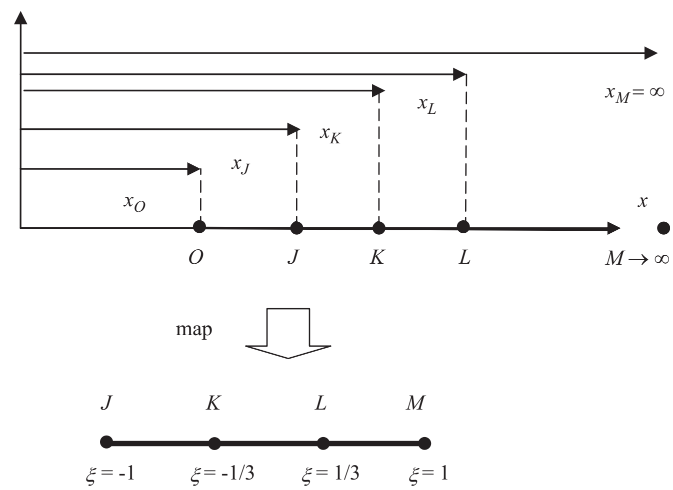

Fig. 3. Proposed mapping of one-dimensional infinite element.

From a practical point of view infinite elements can be classified [2] into five classes:

- classical infinite elements,
- decay infinite elements,
- mapped infinite elements,
- elastodynamic infinite elements and
- wave envelope infinite elements.

The origin of the idea and the development of every one of the above classes are difficult to date. The first class infinite elements are based on the original, the so-called "classical" formulation of the infinite elements. In the decay infinite element formulation decay functions from different mathematical types are used. The mapped infinite elements are developed by using mapping functions. These functions map the infinite domain of the element into a finite domain. Through this approach the obtained infinite element is similar to the classical finite element. The latest research on infinite elements is devoted to the development of elastodynamic infinite elements and wave envelope infinite elements. The last two classes can be treated as a special combination of the mapped and decay infinite elements Fig. 3.

3. Mapping functions

In general, for mapped formulation, the mapping of local to global coordinates along an infinite element domain can be