Finite Elements in Analysis and Design 45 (2009) 624–631

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Relatively simple finite element formulation for the large amplitude free vibrations of uniform beams

R.K. Gupta[a], Gunda Jagadish Babu[a], G. Ranga Janardhan[b], G. Venkateswara Rao[c,*]

[a]Advanced Systems Laboratory, Kanchanbagh, Hyderabad 500058, India
[b]J.N.T.U. College of Engineering, Kakinada 533003, India
[c]Sreenidhi Institute of Science and Technology, Yamnampet, Ghatkesar, Hyderabad 501301, India

ARTICLE INFO

Article history:
Received 28 April 2008
Received in revised form 18 February 2009
Accepted 19 April 2009
Available online 28 May 2009

Keywords:
Free vibration
Large amplitude
Beams
FEM
Iterative solution

ABSTRACT

Large amplitude free vibration analysis of uniform, slender and isotropic beams is investigated through a relatively simple finite element formulation, applicable to homogenous cubic nonlinear temporal equation (homogenous Duffing equation). All possible boundary conditions where the von-Karman type nonlinearity is applicable, where the ends are axially immovable are considered. The finite element formulation begins with the assumption of the simple harmonic motion and is subsequently corrected using the harmonic balance method and is general for the type of the nonlinearity mentioned earlier. The nonlinear stiffness matrix derived in the present finite element formulation leads to symmetric stiffness matrix as compared to other recent formulations. Empirical formulas for the nonlinear to linear radian frequency ratios, for the boundary conditions considered, are presented using the least square fit from the solutions of the same obtained for various central amplitude ratios. Numerical results using the empirical formulas compare very well with the results available from the literature for the classical boundary conditions such as the hinged–hinged, clamped–clamped and clamped–hinged beams. For the beams with nonclassical boundary conditions such as the hinged–guided and clamped–guided, the numerical results obtained, apparently for the first time and are in line with the physics of the problem.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

Large amplitude free vibration analysis of beams considering geometric nonlinearity has been investigated by various researchers using either the analytical or the approximate continuum and numerical methods. Woinowsky-Krieger [1] investigated the problem of the hinged–hinged beams with axially immovable ends using the elliptic integral solution. Evensen [2] studied the nonlinear vibrations of beams using the perturbation method. Srinivasan [3] used the Ritz–Galerkin method to study the nonlinear vibration response of simply supported beams and plates. Ray and Bert [4] used the Ritz–Galerkin method to study the large amplitude free vibrations of beams with pinned ends. The first finite element (FE) formulation for this problem has been developed by Mei [5–7], wherein the axial tensile force developed in the beam is assumed to be constant in an element, but varying from element to element, which is not the true situation. Venkateswara Rao et al. [8,9] formulated the large amplitude free vibrations of beams and plates by linearizing the quadratic terms in the strain displacement relation, and neglected the contribution of the axial displacement. However, the earlier FE formulations gave acceptably accurate results for the uniform slender beams as well as for the thin circular and rectangular plates. Effect of the axial displacement has been considered by Raju et al. [10] and also shown that the linearization of strain–displacement relation amounts to the averaging of the nonlinear stretching force in the beam.

Prathap and Bhashyam [11], and Sarma and Varadan [12] concluded that the above discussed formulations are incorrect, since the axial displacement is neglected and the axial stretching force is averaged. In all the above formulations the assumptions of the simple harmonic motion (SHM) is used and as a result the equation of motion is satisfied only at the instant of maximum amplitude.

Kapania and Raciti [13] studied the nonlinear free vibrations of the composite beams. In this formulation they reduced the dynamic FE matrix equation to a scalar equation by using the linear mode shapes obtained with the assumption of the SHM. The scalar equation thus obtained was solved by the perturbation method to obtain the nonlinear to linear frequency ratios. However, in this formulation the out-of-plane equilibrium equations are not exactly satisfied. Singh et al. [14] studied the problem by solving iteratively the

* Corresponding author.
E-mail address: hydrao1944@yahoo.co.in (G. Venkateswara Rao).

0168-874X/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.finel.2009.04.001

Nomenclature

a	central amplitude of the vibrating beam
A	cross-section area of beam
E	Young's modulus
I	area moment of inertia
$[k_e]$	element stiffness matrix
$[K]$	assembled stiffness matrix
l	element length
L	length of the beam
m	mass per unit length
$[m]$	element mass matrix
$[M]$	assembled mass matrix
r	radius of gyration ($r=\sqrt{I/A}$)
u	axial displacement
U	strain energy
w	lateral displacement
T	kinetic energy
x	axial coordinate
$\alpha_1 \rightarrow \alpha_8$	generalized coordinates
ε_x	axial strain
λ	eigenvalue (frequency parameter $=m\omega^2L^4/EI$)
ψ_x	curvature
ω	radian frequency
δ	eigenvector (mode shape of vibration)

Subscripts

L	linear
NL	nonlinear
H	harmonic

Superscript

$(\ )'$	differentiation with respect to x
$[\ ]^T$	transpose of a matrix

dynamic finite element matrix equation such that the equations corresponding to the axial and out-of-plane directions are exactly satisfied. The converged eigenvector obtained using the SHM assumption is used for reducing the dynamic FE matrix equation iteratively to a scalar homogenous cubic nonlinear (homogenous Duffing equation), which is solved by the direct numerical integration (DNI). In this formulation the final stiffness matrix is unsymmetric due to the coupling of the axial and transverse displacement, and the solution of the eigenvalue problem involving unsymmetric matrix is not that attractive when compared to the symmetric one as very efficient and foolproof algorithms are available to solve the eigenvalue problem containing symmetric matrices [19].

Chen et al. [20] studied the nonlinear vibration of plane structures using the finite element and incremental harmonic balance (IHB) method. Note that the IHB method presented in Ref. [20] is a combination of incremental method (Newton–Raphson method) with the harmonic balance method (HBM, Ritz–Galerkin averaging method). This formulation is applied to predict the fundamental resonance, super and subharmonic response and combination of resonance of plane structures. Leung et al. [21] presented a computational algorithm to construct the back-bone curve of an elastic body in large amplitude vibration in which the amplitude of the structure is expressed in terms of harmonic components. In this formulation the increment is applied to the harmonic coefficients of the displacement amplitudes to find new equilibrium states along the back-bone curve. The researchers [20,21] primarily studied the amplitude–frequency response of the structures.

Researchers in Refs. [22–27] discussed the field consistent strain formulations of the displacement based finite element formulation where authors [26] emphasized that if the assumed strain field are not variationally correct FE procedure can lead to poor convergence and spurious stress oscillations. Reddy [22] discussed the possible locking effects arising in beams due to the assumed inconsistent displacement field variation when the nonlinear strain–displacement relations of von-Karman type are considered.

In the present study, the large amplitude free vibration analysis of the uniform, slender and isotropic beams is investigated with all possible boundary conditions on w and both the ends of the beam constrained to move axially, resulting in von-Karman type strain–displacement relation. The present formulation considers the nonlinear strain–displacement relation without any approximation. Note that there are a number of continuum and FE formulations where the axial deformation is considered indirectly using the tension developed in the beam due to large deflections. In some formulations the axial deformation is neglected and in other the axial deformation is directly considered as in the present formulation. An exhaustive study is carried out for the hinged–hinged (H–H), clamped–clamped (C–C), clamped–hinged (C–H), clamped–guided (C–G) and hinged–guided (H–G) beams starting with the SHM assumption. The guided boundary condition is of two types and are denoted by G1 and G2, where G1 represents that the lateral displacement and the rotation are not constrained and in G2, there is no constraint to lateral displacement while the rotation is constrained. The final solution in terms of ratios of the nonlinear to linear radian frequencies for several central amplitude ratios are obtained by applying the harmonic balance method [16] to correct the error involved in the assumption of the SHM for the abovementioned boundary conditions. It may be emphasized that the matrices involved in the eigenvalue problem are symmetric in the present FE formulation. Numerical results for the classical boundary conditions H–H, C–C and C–H beams are available in the literature and the present results compare very well with those and at the same time the corresponding results for the nonclassical boundary conditions C–G1, C–G2, H–G1 and H–G2 beams are not readily available and are presented perhaps for the first time. The simplicity of the present FE formulation lies in getting the realistic solution using the HBM [16] to correct for the assumption of the SHM contrary to the procedures followed in the Refs. [13,14].

2. Finite element formulation

The nonlinear strain–displacement relation of the beam with the axially immovable ends are given by

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \tag{1}$$

and

$$\psi_x = \frac{d^2w}{dx^2} \tag{2}$$

Note that Eq. (1) is valid for small strain but moderately large rotation and transverse deflection (of the order of characteristic dimension of the cross-section) of the beam [22,28].

The beam is divided into a number of finite elements of equal length l. The strain energy (U) of the element is

$$U = \frac{EA}{2}\int_0^l \varepsilon_x^2\,dx + \frac{EI}{2}\int_0^l \psi_x^2\,dx \tag{3}$$

Download English Version:

https://daneshyari.com/en/article/514142

Download Persian Version:

https://daneshyari.com/article/514142

Daneshyari.com