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Response sensitivity and Hessian matrix analysis of planar frames subjected to earthquake excitation

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A B S T R A C T

This paper describes a new method of calculating accurately and efficiently dynamic response sensitivity and Hessian matrix of planar frames subjected to earthquake excitation. The formulas for sensitivity and Hessian matrix are derived by direct differentiation. An efficient algorithm to calculate dynamic response sensitivity and Hessian matrix is formulated based on finite element method and Newmark-β method. The first and second derivatives of dynamic responses can be derived simultaneously with only a single dynamics analysis. Two numerical examples are demonstrated with the newly developed method and the central-difference method. The results show that compared with the central-difference method, the new method proposed in this paper is highly accurate and more efficient.

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1. Introduction

Response sensitivity analysis of structures subjected to dynamic loading deals with calculation of first derivatives of dynamic response with respect to the design variables, and response Hessian matrix analysis deals with calculation of second derivatives of dynamic response with respect to the design variables. These derivatives are often used in the solution of various problems. In structural optimal design, they are often required to select a search direction in some mathematical programming methods, e.g. Newton's Method and Quasi-Newton's Methods [1]. Many methods of calculating dynamic response sensitivity have been developed [2–5]. Accuracy and efficiency of these methods are also discussed in many studies [6–8]. Methods of dynamic response sensitivity for discretized systems are mainly divided into three categories [9]: finite-difference methods, analytical methods and "semi-analytical" methods. Although many researchers have calculated dynamic response sensitivity, there is little work published on dynamic response Hessian Matrix analysis. Dynamic response Hessian Matrix analysis, i.e., second derivatives of dynamic response with respect to the design variables, is more difficult and more complicated than the sensitivity analysis and therefore computationally expensive. However, the efficiency of the structural optimization using both first derivative and second derivative can be greatly improved when the gradient and Hessian matrix can be calculated accurately and efficiently.

The purpose of this paper is to develop a new method for calculating accurately and efficiently dynamic response sensitivity and Hessian matrix of planar frames subjected to earthquake excitation. The new method is based on the finite element method and the Newmark-β method [10]. The dynamic responses include node displacements, node accelerations, node velocities, interstory drifts, node forces and cross-sectional internal forces at the middle of element. The paper is arranged as follows. In Section 2, the first and second derivatives of planar frame stiffness, mass and damping matrix with respect to structural design variables are calculated based on the finite element method. In Section 3, the formulas for dynamic response sensitivity and Hessian matrix are derived by direct differentiation. An algorithm to calculate dynamic response sensitivity and Hessian matrix is formulated based on the finite element method and the Newmark-β method. In Section 4, formulas for the dynamic response sensitivity and Hessian matrix using the central-difference method are presented. Finally, two numerical examples are demonstrated with the newly developed method and the central-difference method. Compared to the central-difference method, the numerical examples show that the new method presented in this paper is sufficiently accurate and more efficient.

2. First and second derivatives of structural stiffness, mass and damping matrices

In order to calculate dynamic response sensitivity and Hessian matrix, first and second derivatives of the total structural stiffness,

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mass, and damping matrices with respect to the structural design variables have to been obtained. In this work, the first and second derivatives of the planar frame element stiffness matrix and mass matrix are calculated first. Then the first and second derivatives of the planar frame element stiffness matrix and mass matrix are assembled to obtain the first and second derivatives of the total stiffness matrix and mass matrix of frame. The assembly process is similar to the one in finite element method, i.e., the element stiffness matrix and mass matrix are assembled to obtain the total structural stiffness matrix and mass matrix. The first and second derivatives of the structural damping matrix with respect to structural design variables are calculated according to Rayleigh damping hypothesis.

Nodal displacement of a planar frame element is shown in Fig. 1 and nodal force in Fig. 2. Element number is denoted by e. Local and global coordinate system are $\bar{x}0\bar{y}$ and $x0y$, respectively. $\bar{x}$ is assumed to be directed from node i to node j. Angle between the global coordinate x and the local coordinate $\bar{x}$ axes is φ (clockwise). Element length is denoted by l_e. Cross section of element is rectangle (or other cross-section styles). Width b_e and height h_e of the rectangle section shown in Fig. 3 are defined as design variables of element e.

Nodal displacement vector of element e in a local coordinate is defined as

$$\bar{\boldsymbol{\delta}}^e = [\bar{u}_i \quad \bar{v}_i \quad \bar{\theta}_i \quad \bar{u}_j \quad \bar{v}_j \quad \bar{\theta}_j]^T \tag{1}$$

and in a global coordinate is

$$\boldsymbol{\delta}^e = [u_i \quad v_i \quad \theta_i \quad u_j \quad v_j \quad \theta_j]^T \tag{2}$$

where superscript T is vector transpose and subscript is node number of element. Angle displacements $\theta_i = \bar{\theta}_i$ and $\theta_j = \bar{\theta}_j$.

Nodal force vector of element e in a local coordinate is defined as

$$\bar{\boldsymbol{F}}^e = [\bar{X}_i \quad \bar{Y}_i \quad \bar{M}_i \quad \bar{X}_j \quad \bar{Y}_j \quad \bar{M}_j]^T \tag{3}$$

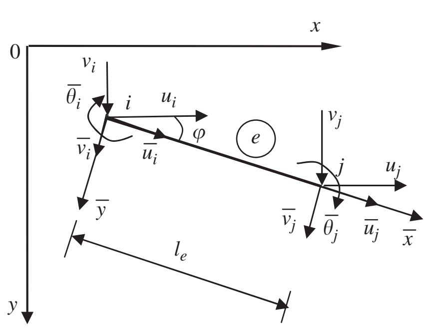

Fig. 1. Beam element node displacements.

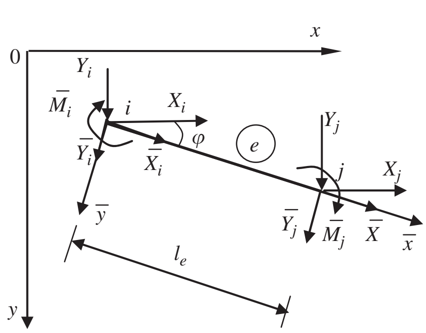

Fig. 2. Beam element node forces.

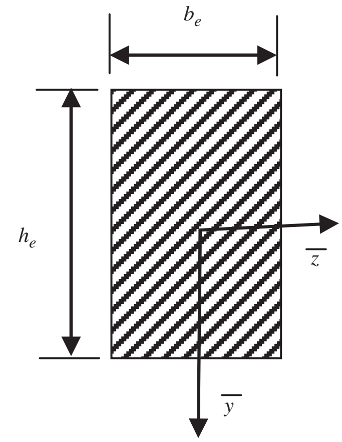

Fig. 3. Element design variables.

and in a global coordinate:

$$\boldsymbol{F}^e = [X_i \quad Y_i \quad M_i \quad X_j \quad Y_j \quad M_j]^T \tag{4}$$

where bending moments $M_i = \bar{M}_i$ and $M_j = \bar{M}_j$.

2.1. First and second derivatives of structural total stiffness matrix

Element stiffness matrix in a local coordinate system can be expressed with element design variables (i.e., width b_e and height h_e):

$$\bar{\boldsymbol{K}}^e = \begin{bmatrix} \frac{Eb_eh_e}{l_e} & 0 & 0 & \frac{-Eb_eh_e}{l_e} & 0 & 0 \\ 0 & \frac{Eb_eh_e^3}{l_e^3} & \frac{Eb_eh_e^3}{2l_e^2} & 0 & \frac{-Eb_eh_e^3}{l_e^3} & \frac{Eb_eh_e^3}{2l_e^2} \\ 0 & \frac{Eb_eh_e^3}{2l_e^2} & \frac{Eb_eh_e^3}{3l_e} & 0 & \frac{-Eb_eh_e^3}{2l_e^2} & \frac{Eb_eh_e^3}{6l_e} \\ \frac{-Eb_eh_e}{l_e} & 0 & 0 & \frac{Eb_eh_e}{l_e} & 0 & 0 \\ 0 & \frac{-Eb_eh_e^3}{l_e^3} & \frac{-Eb_eh_e^3}{2l_e^2} & 0 & \frac{Eb_eh_e^3}{l_e^3} & \frac{-Eb_eh_e^3}{2l_e^2} \\ 0 & \frac{Eb_eh_e^3}{2l_e^2} & \frac{Eb_eh_e^3}{6l_e} & 0 & \frac{-Eb_eh_e^3}{2l_e^2} & \frac{Eb_eh_e^3}{3l_e} \end{bmatrix} \tag{5}$$

where E and l_e are material elastic modulus and element length, respectively. The coordinate transformation matrix $\boldsymbol{T}$ of element e is defined by

$$\boldsymbol{T}^e = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 & 0 & 0 & 0 \\ -\sin\varphi & \cos\varphi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\varphi & \sin\varphi & 0 \\ 0 & 0 & 0 & -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$\bar{\boldsymbol{K}}^e$ and $\boldsymbol{T}^e$ are obtained from Eqs. (5) and (6), respectively, the stiffness matrix of element e in a global coordinate is given by

$$\boldsymbol{K}^e = \boldsymbol{T}^{eT}\bar{\boldsymbol{K}}^e\boldsymbol{T}^e \tag{7}$$

First and second derivatives of stiffness matrix of element e with respect to the structural design variables (note: the structural design variables consist of all the element design variables) are obtained by differentiating Eq. (7) with respect to the design variables, i.e., width b_e and height h_e of each element. Obviously, if $e \neq k$, the first and second derivatives of stiffness matrix of element e with respect

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