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Coupled electrothermal–mechanical analysis for MEMS via model order reduction

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A R T I C L E I N F O

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This paper investigates model order reduction (MOR) techniques that can be used in conjunction with finite element schemes to generate computationally efficient solutions for multiphysics MEMS simulation. The Lanczos and Arnoldi algorithms are implemented to extract low dimensional Krylov subspaces from the finite element discretized system for model order reduction. A deflation procedure is employed in both algorithms to improve the solution convergence of the implicit iterations together with stopping criteria to automatically determine the reduced model size. Reduced order electro-thermal–mechanical models are generated for a MEMS microgripper using the developed programs. A Guyan-based ANSYS substructuring analysis of the same device is also performed. Results discussion on all three techniques including preservation of full scale model properties such as dynamic behavior and stability are presented along with comparisons of reduced and full model simulation. The developed programs automatically generate compact structure preserving models and can be used to significantly improve the computational efficiency without much loss in accuracy and model stability for coupled-field MEMS simulation.

1. Introduction

Microelectromechanical systems (MEMS) have attracted much attention over the past decades [1,2]. Among various successful commercial MEMS applications, actuators have become a growing topic of interest. MEMS actuators are integrated devices that generate motion by coupled physics field's interactions, typically through piezoelectric, electrostatic or electromagnetic effects. Examples of such MEMS can be found in inkjet printer heads, scanning probe microscopes, miniature mechanical switches and game controllers [3–5]. Electrothermal actuators present a class of conventional MEMS devices with great potential for science and applications. They have been shown to exhibit high sensitivity and are widely used in bio-sampling, optical switching and micro-positioning applications [6–8].

Finite element analysis (FEA) has been commonly used to simulate multiphysics effects of MEMS structures [9–11]. However, MEMS designers are continually reaching FEA limits due to excessive demands on CPU-time and memory resources [12]. Advances in microfabrication techniques have allowed the creation of many complex MEMS devices. The increase in design complexity and the coupling in multiple physical domains have posed great challenges in MEMS simulation. Computationally demanding finite element analyses are constantly needed to achieve accurate results for multiphysics MEMS. When transient or dynamic MEMS behaviors are considered, the amount of computation during finite element analysis for time-history results is often a daunting task.

Model order reduction (MOR) techniques [12–15,22] have been presented to generate computationally efficient solutions by replacing a large-scale discretized model with a reduced model able to characterize the dynamic behavior and preserve essential model properties. An early method proposed by Guyan [15] generates a reduced model by considering dominant degrees of freedom (DOFs) of the original model and neglecting all others. The choice of dominant DOFs is based on intuition and experience resulting in a highly sensitive reduction method. Another common method is balanced model truncation [16] which is based on the removal of uncontrollable or unobservable eigenmodes. Various studies [17,18] have implemented balanced truncation to create reduced models. However, this method involves solution of the computationally expensive Lyapunov equations [19].

Recent years has seen an increased use of Krylov subspace reduction methods. These methods lack the computational cost of balanced model truncation while requiring minimal user intervention as opposed to Guyan [20–24]. Krylov methods construct a reduced model based on either explicitly or implicitly approximating the transfer function of a full scale model.

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Asymptotic waveform evaluation (AWE) is an explicit method that has been used to create reduced models [20,21]; however, AWE has been shown to be a numerically unstable process [22].

The Arnoldi and Lanczos algorithms [22–24] are implicit methods that generate Krylov subspaces and reduce models by projection onto these subspaces. Researchers [22,23] have simulated large dynamic systems via two-sided Lanczos projection. A drawback to this method is the reduced solution describes the response for a single node, in addition, loss of stability may occur [22,23]. Bai and Freund [24] presented a modified version of Lanczos where stability is guaranteed. However, this trial and error method has been shown to affect reduced model accuracy [24]. Arnoldi has also been the subject of a number of recent multiphysics studies [25–29]. The authors of [25] reviewed model reduction techniques including Arnoldi followed by presentation of reduced order modeling of electrostatically actuated microbeams and microplates. Researchers [26] presented Arnoldi reduction for two benchmark applications. The discussion was limited to electrostatic actuation and electrothermal coupling. Rudnyi's group [27–29] has done a lot of work in the area of reduced order modeling of electrothermally actuated MEMS. Arnoldi model reduction was presented in [27] where a MEMS gas sensor was simulated to obtain the transient temperature distribution throughout the device due to electric resistance heating. A review on electrothermal modeling of microsystems was provided in [28,29] with discussions of Krylov subspace projection via the Arnoldi and Lanczos algorithms. It was pointed out in [29] that a major disadvantage of the Krylov-based Arnoldi and Lanczos algorithms is the lack of a global error estimation necessitating the manual choice of the reduced models order for solution convergence. To the best of our knowledge, the previous MOR studies in electrothermal MEMS have only considered electrothermal domain coupling.

In this study, electrothermal MEMS modeling via MOR is extended to electrothermal–mechanical modeling. In addition, an attempt is made to use a deflation procedure to improve the solution convergence. The paper is organized as follows. In Section 2 the electrothermal–mechanical governing differential equations and finite element discretization is laid out yielding a coupled system of ordinary differential equations (ODEs). Section 3 examines model order reduction of a system of ODEs beginning with a description of moment matching and Krylov subspace reduction followed by presentation of the Lanczos and Arnoldi algorithms. Model order reduction simulation results of an electrothermally actuated microgripper are reported throughout Section 4 followed by a discussion of results including preservation of full scale model properties along with a comparison of reduced and full model analysis in Section 5 and concluding remarks are in Section 6.

2. Coupled electrothermal–mechanical formulation

Electrothermal actuators operate based on the interactions of the electric, thermal and mechanical domains. While these interactions occur simultaneously, minimal effect of thermal induced deformation on the thermal domain allows for a sequential coupling modeling approach where the temperature solution of the electrothermal domain is distributed throughout mechanical domain to obtain the devices deformation. When FEA is used for solution, sequential coupling can reduce the problem size which can be advantageous in design iterations. In electrothermal coupling, resistive heating or the so called Joule heating is the dominant internal heat generation mechanism [27]. Assuming uniform internal heat generation over a lumped resistor allows for the decoupling of the electrical and thermal domains. Under this assumption, the temperature distribution throughout an electrothermal device is governed by the following heat transfer equation [27]:

$$\nabla(k\nabla T)+Q=\rho C_p\frac{\partial T}{\partial t} \quad \text{with} \quad Q=\frac{V^2}{R} \tag{1}$$

where T is the temperature, k the thermal conductivity, Q the internal heat generation, ρ the density, C_p the specific heat, V the applied voltage and R the electrical resistance.

Finite element discretization of Eq. (1) yields

$$C\dot{T}+KT=PQ \tag{2}$$

where C is the heat capacitance matrix, K the conductance matrix, and P is a vector that describes Q's distribution throughout the device.

The non-uniform temperature distribution obtained through solution of Eq. (2) produces thermal stresses due to thermal expansion. Coupling of the thermal domain with the mechanical domain results in the following discretized structural equation [30]:

$$M^u\ddot{d}+C^u\dot{d}+K^ud=F^{th} \tag{3}$$

where M, C and K are the mass, damping and stiffness matrices, respectively, with the superscript u denoting the mechanical domain, d is the nodal displacement vector, and F^{th} the induced thermal loading.

Derivation of system matrices in Eqs. (2) and (3) can be made following standard finite element discretization procedures [30]. Taking Eq. (3) as an example, the structural matrices can be obtained based on the principle of virtual work for a volume element V with surface area S as follows:

$$\int_V((\partial\sigma)^T\rho\ddot{u}+(\partial u)^Tb\dot{u}+(\partial\varepsilon)^T\sigma)dV=\int_V((\partial\varepsilon)^T\sigma^{th})dV \tag{4}$$

where ρ, b, σ and σ^{th} are the mass density, damping coefficient, mechanical stress and thermal stress, respectively; ∂u and $\partial\varepsilon$ represent virtual displacements and their corresponding strains. Finite element discretization gives:

$$u=Nd \quad \text{and} \quad \varepsilon=Bd \tag{5}$$

where N is the shape function matrix and B the strain–displacement matrix. Combination of Eqs. (4) and (5) results in

$$(\partial d)^T[m\ddot{d}+c\dot{d}+kd]=(\partial d)^Tf^{th} \tag{6}$$

where$m=\int_V N^T\rho N\,dV$, $c=\int_V N^TbN\,dv$, $k=\int_V B^TEB\,dV$ and $f^{th}=\int_V B^TE\varepsilon^{th}dV$, ε^{th} and E represent the thermal strain and the elasticity matrix, respectively. Assembling the above element formulation will lead to its global form as shown in Eq. (3) which along with Eq. (2) forms a coupled global system of ODEs for modeling electrothermal MEMS devices.

3. Model order reduction

Model order reduction methods have demonstrated desirable model size reduction [27–29]. However, to gain widespread use in MEMS simulation, MOR implementations are needed that require minimal user intervention while preserving original model characteristics. Two Krylov projection-based techniques, Lanczos and Arnoldi methods, are discussed in this section. Krylov subspace projection methods extract low dimensional Krylov subspaces from models described by ODEs with the desired model reduction achieved by projection of the models onto the subspaces while dynamic characteristics are maintained through a property called moment matching. To illustrate the MOR formulation procedure and moment matching we'll take the first order system shown in Eq. (2) as an example and rewrite it as