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Topology optimization of frequency responses of fluid–structure interaction systems

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ABSTRACT

This paper presents a topology optimization method for the frequency response of a multiphysics system involving fluid–structure interaction. The finite element analysis of the system is carried out based on the $\mathbf{u}_s/p_f$ formulation. The structural domain is governed by the linear equation of elasticity and expressed in terms of the displacement, $\mathbf{u}_s$, and the fluid domain is described by Helmholtz equation via the primary variable of pressure, p_f. The coupling conditions are the equilibrium and kinematic compatibilities at the fluid–structure interface. The optimization procedure used in this work is based on the bi-directional evolutionary structural optimization (BESO). Due to the binary characteristics of the BESO method of adding/removing material, the methodology proposed here circumvents some problems faced by the traditional density based optimization methods, especially concerning the fluid–structure interface during the optimization process. The proposed methodology can be applied to various engineering problems such as noise reduction in passenger compartments in automobiles and aircraft, and vibration control of submerged structures. Several numerical examples are presented demonstrating that the proposed BESO method can be used for the topology optimization of these kinds of multiphysics problems effectively and efficiently for 2D and 3D cases.

1. Introduction

In the past few decades topology optimization has been used to improve the characteristics and increase the performance of a wide variety of systems. The basic idea is to find an optimal distribution of material in a structural design domain, considering the objective function and design constraints. A number of different topology optimization approaches have been developed including the homogenization method [1], the solid isotropic material with penalization (SIMP) method [2,3], the evolutionary structural optimization (ESO) method [4,5], and the level set method [6,7].

Topology optimization has been extended to various dynamic design problems. Among early works of topology optimization considering dynamic response are those by Diaz and Kikuchi [8], Ma et al. [9,10] and Xie and Steven [11,12]. Since then, many other studies have been carried out to develop topology optimization techniques for dynamic problems [13–22].

Although structural optimization involving dynamic response has been investigated by many researchers, only a few have explored the frequency response problems involving fluid–structure interaction systems, e.g. Duhring et al. [23], Akl et al. [24], Yoon [25,26], Niu et al. [27], Zhang et al. [28,29] and Shu et al. [30]. Optimization of the frequency response of fluid–structure systems is of great importance in many practical engineering problems such as noise reduction in automobiles and aircraft, fatigue of offshore structures, and performance of musical instruments.

Du and Olhoff [31,32] have considered vibrating structures in a surrounding acoustic medium and conducted the topological design with respect to optimum sound pressure characteristics. In these works, the elastic structure is placed in an acoustic medium and both structural and acoustic domains do not change their locations. However, when coupled problems are considered and the acoustic-structure interfaces can change their locations during the optimization process, the classical density based topology optimization methods become arduous [25]. In these methods, the interfaces are not explicitly defined due to the existence of intermediate density elements and the coupling boundary conditions cannot be modelled straightforwardly. In order to circumvent this problem, Yoon et al. [25] have proposed mixed finite elements, where the structural and acoustic domains are overlapped and the acoustic medium is approximated by equating the elastic shear modulus to zero. More recently, Shu et al. [30] have extended the level set based topology optimization to a class of acoustic-structural problems, in

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order to minimize the response (pressure/displacement) of the coupled systems.

In this context, the aim of this work is to develop a topology optimization procedure to minimize frequency responses of coupled fluid–structure systems using the evolutionary topology optimization methodology. An alternative approach to topology optimization of acoustic-structural interaction systems using an extended BESO method [33,34] for the frequency response function is presented. Previous works on ESO/BESO methods for dynamic problems have been mainly focused on the frequency optimization [12,35–38].

In the present work, the classical displacement–pressure ($\mathbf{u}_s/p_f$) formulation [39,40] is used for the analysis of the fluid–structure interaction. The solid domain is governed by the elasticity equation and the fluid domain by Helmholtz equation. The two separate fields are fully coupled by the surface-coupling integral in each step of the optimization process, which guarantees equilibrium conditions on the acoustic-structural interface. The main advantage of this $\mathbf{u}_s/p_f$ formulation is the reduced number of the degree of freedom necessary to describe the fluid domain – only one is required per node. For the three-dimensional cases this feature leads to a significant saving on the computational time. This formulation has as primal variables the displacement, $\mathbf{u}_s$, and the pressure, p_f, which are the same variables of interesting in the optimization process. Detailed descriptions of this formulation and various other for coupled fluid–structure systems can be found in the articles contained in Sandberg and Ohayon [41].

This paper is organized as follows: Section 2 presents the governing equations and the finite element model for the acoustic-structural coupled system. In Section 3, the topology optimization problem for the frequency response minimization is formulated and the sensitivity analysis is carried out. Section 4 shows the numerical results achieved with the proposed methodology. Finally, conclusions are drawn in Section 5.

2. Fluid–structure interaction: governing equations and finite element model

A sketch of the coupled fluid–structure system to be optimized in this work is shown in Fig. 1, where Ω_s is the structure domain; Ω_f is the fluid domain; Γ_{sf} is the interface between the domains; $\mathbf{n}_s$ and $\mathbf{n}_f$ are the boundaries unit normal vectors pointing outward from the structural and fluid domain, respectively; $\mathbf{n}$ is the unit normal vector at the interface pointing outward from the fluid domain.

2.1. Equilibrium equations

Some assumptions are made in order to model the coupled fluid–structure system [42,43]. The structural part of the system is considered to be homogeneous, isotropic and undertaking small deformation. Thus, the classical linear elastodynamic equation for a continuum medium is used:

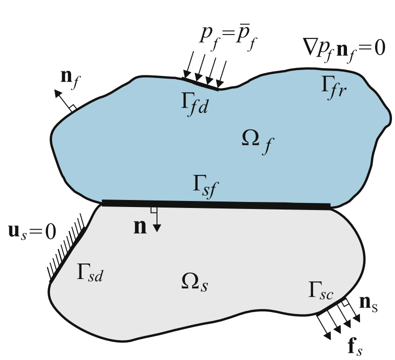

Fig. 1. Coupled fluid–structure system.

$$\nabla \cdot \boldsymbol{\sigma}_s - \rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} = 0 \quad \text{in } \Omega_s \tag{1}$$

where $\nabla \cdot \boldsymbol{\sigma}_s$ is the divergence of the Cauchy stress tensor; $\mathbf{u}_s$ is the displacement vector field of the structure and ρ_s is the mass density of the structure.

In this paper the fluid medium is considered to be inviscid, irrotational and having only small translation, the wave equation for the medium can be derived in terms of the sound pressure:

$$\frac{1}{c_0^2}\frac{\partial^2 p_f}{\partial t^2} - \nabla^2 p_f = 0 \quad \text{in } \Omega_f \tag{2}$$

where p_f is the pressure scalar field and c_0 is the constant speed of sound in the fluid.

2.2. Boundary conditions

In the structural domain, the Dirichlet boundary condition is considered:

$$\mathbf{u}_s = 0 \quad \text{in } \Gamma_{sd} \tag{3}$$

and the Neumann boundary conditions:

$$\boldsymbol{\sigma}_s \mathbf{n}_s = \mathbf{f}_s \quad \text{in } \Gamma_{sc} \tag{4}$$

$$\boldsymbol{\sigma}_s \mathbf{n}_s = p_f \mathbf{n}_f \quad \text{in } \Gamma_{sf} \tag{5}$$

Eq. (3) represents the structural displacement constraint applied to the structure; Eq. (4) is the prescribed external load, $\mathbf{f}_s$, applied to the structure; Eq. (5) indicates the action of pressure forces exerted by the fluid on the structure and represents the equilibrium condition at the interface between the domains.

In the fluid domain, the following Dirichlet boundary condition is considered:

$$p_f = \bar{p}_f \quad \text{in } \Gamma_{fd} \tag{6}$$

and the Neumann boundary conditions:

$$\nabla p_f \mathbf{n}_f = 0 \quad \text{in } \Gamma_{fr} \tag{7}$$

$$\nabla p_f \mathbf{n}_f = \rho_f \frac{\partial^2 \mathbf{u}_s}{\partial t^2}\mathbf{n}_s \quad \text{in } \Gamma_{sf} \tag{8}$$

where ρ_f is the mass density of the fluid medium.

Eq. (6) is the prescribed pressure applied to fluid; Eq. (7) expresses the rigid wall boundary condition; Eq. (8) represents the kinematic compatibility of the normal displacements at the interface of the fluid and structural domains.

2.3. Finite element discretization

In order to obtain the finite element discretization of the system, the weak form of the differential equilibrium equations is derived [44–46].

In the weighted residual approximation of the structural domain, Eq. (1) is multiplied by a set of weight functions, $\mathbf{w}_s$, and integrated over the structural domain Ω_s

$$\int_{\Omega_s} \mathbf{w}_s^T \left(\nabla \cdot \boldsymbol{\sigma}_s - \rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} \right) dV = 0 \tag{9}$$

Using Green's theorem, the weak form of the equilibrium equation for the structural domain can be written as

$$\int_{\Omega_s} (\mathbf{w}_s)^T \rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} dV + \int_{\Omega_s} (\nabla \mathbf{w}_s)^T \boldsymbol{\sigma}_s \, dV = \int_{\Gamma_{sf}} (\mathbf{w}_s)^T p_f \mathbf{n}_f \, dS + \int_{\Gamma_{sc}} (\mathbf{w}_s)^T \mathbf{f}_s \, dS \tag{10}$$