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Co-rotational finite element formulation used in the Koiter–Newton method for nonlinear buckling analyses

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ABSTRACT

The Koiter–Newton approach is a novel reduced order modeling technique for buckling analysis of geometrically nonlinear structures. The load carrying capability of the structure is achieved by tracing the entire equilibrium path in a stepwise manner. At each step a reduced order model generated from Koiter's asymptotic expansion provides a nonlinear prediction for the full model, corrected by a few Newton steps. The construction of the reduced order model requires derivatives of the strain energy with respect to the degrees of freedom up to the fourth order, which is two orders more than traditionally needed for a Newton based nonlinear finite element technique. In this paper we adopt the co-rotational formulation to facilitate these complex differentiations. We extend existing co-rotational beam and shell element formulations to make them applicable for the high order derivatives of the strain energy. The geometrical nonlinearities are taken into account using derivatives of the local co-rotational frame with respect to global degrees of freedom. This is done outside the standard element routines and is thus independent of the element type. We utilize three configurations and the nonlinear rotation matrix to describe finite rotations of the shell accurately, and profit from the automatic differentiation technique to optimize the programming of high order derivatives. The performance of the proposed approach using the co-rotational formulation is demonstrated using benchmark examples of isotropic and laminated composite structures.

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1. Introduction

The analysis of geometrically nonlinear responses of structures is important for determining their load carrying capability, especially in the case of buckling where a limit point or a bifurcation point exists [1–8]. Utilizing the expanding computational power of modern computers nonlinear finite element analysis (FEA) has become the standard technique used to obtain the nonlinear response of complex structures, however, the repeated analyses that are needed for FEA in the design loop are still computationally demanding. Thus, reduced order techniques that can be used to reduce a problem's size significantly are attractive.

Koiter reduction methods [9–17] use the Koiter's famous perturbation technique [18] to reduce the number of degrees of freedom in the finite element model. The main advantage of Koiter's theory [2,18–20] is the capability to predict quickly but accurate enough the propensity of a structure to buckle and to provide the structure's initial postbuckling behavior. In Koiter's perturbation technique, his asymptotic expansion is used only once at the bifurcation point to construct the reduced order model (ROM) which can present the initial postbuckling path of the structure. Hence, the traditional Koiter reduction method is valid asymptotically in the neighborhood of the bifurcation point. The majority of research done in this field applies an expansion of the displacement field up to the second order which is usually accurate enough to capture the initial postbuckling response of structures [16,21–24]. Damil and Potier-Ferry [25] have adopted higher order terms to increase the range of validity of the perturbation expansion further. Increasing the number of higher order terms in the displacement provides a wider range of validity. Yet, the reduced order model obtained from a single perturbation expansion still has a limited range of validity that cannot be determined a priori. In addition, the prebuckling state is assumed to be linear in most of Koiter reduction methods used in [10,15,16,24,26,27], since these methods are based on an alternative procedure proposed in Budiansky and Hutchinson [28], in which the assumption

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Nomenclature

List of symbols

N	number of degrees of freedom in the full finite element model
$1+m$	number of degrees of freedom in the reduced order model
m	number of closely spaced buckling modes
$\mathbf{f}_{ext}$, $\mathbf{f}_{int}$, $\mathbf{f}_r$	external load, internal load and residual force
$\mathbf{q}$, $\mathbf{q}_0$	displacements in current and nominal configurations
$\mathbf{u}$	relative displacement from nominal configuration to current configuration
λ, λ_0	load parameters at current and nominal configurations
$\Delta\lambda$	increment of load parameter from nominal to current configurations
$\mathcal{L}$, $\mathcal{Q}$, $\mathcal{C}$	linear, quadratic and cubic forms in expansion of equilibrium equations
L, Q, C	2D, 3D and 4D tensors of $\mathcal{L}$, $\mathcal{Q}$, $\mathcal{C}$
F, $\boldsymbol{\phi}$, $\mathbf{f}_\alpha$	load matrix, load amplitude and sub-loads
$\boldsymbol{\xi}$, $\mathbf{u}_\alpha$, $\mathbf{u}_{\alpha\beta}$, $\mathbf{u}_{\alpha\beta\gamma}$	generalized displacement and first to third order displacement fields
$\overline{\mathcal{L}}$, $\overline{\mathcal{Q}}$, $\overline{\mathcal{C}}$	linear, quadratic and cubic forms in reduced order model
$\overline{\mathsf{L}}$, $\overline{\mathsf{Q}}$, $\overline{\mathsf{C}}$	2D, 3D and 4D tensors of $\overline{\mathcal{L}}$, $\overline{\mathcal{Q}}$, $\overline{\mathcal{C}}$
K_t	tangent stiffness matrix
$\mathbf{E}_\alpha$	α-th unit vector
$\overline{\mathbf{L}}_\alpha$, $\overline{\mathbf{Q}}_{\alpha\beta}$, $\overline{C}_{\alpha\beta\gamma\delta}$	column vectors of tensors $\overline{\mathsf{L}}$, $\overline{\mathsf{Q}}$, components of tensor $\overline{\mathsf{C}}$
$q_{,\lambda}$	linear displacement of the structure under the external load $\mathbf{f}_{ext}$

Symbols used in co-rotational beam element

a, b	node number
(x,y)	global coordinate system
(e_x^0, e_y^0)	co-rotational frame in reference configuration
(e_x, e_y)	co-rotational frame in nominal configurations
$\mathbf{q}$, $\widehat{\mathbf{q}}$	global and local degrees of freedom
$u_a, v_a, \theta_a, u_b, v_b, \theta_b$	components in global degrees of freedom $\mathbf{q}$
$\widehat{u}_b, \widehat{\theta}_a, \widehat{\theta}_b$	components in local degrees of freedom $\widehat{\mathbf{q}}$
l, l_n	initial and current length of beam
θ	angle between nominal configuration and x-axis
θ_0	angle between reference configuration and x-axis
$\Delta\theta$	angle between reference and nominal configurations
$\mathbf{r}_a$, $\mathbf{r}_b$	position vectors of nodes a and b in reference configuration
$\mathbf{d}_a$, $\mathbf{d}_b$	position vectors of nodes a and b in nominal configuration
κ, A, $\overline{U}$	curvature, area of cross section and strain energy
$\mathbf{f}$, $\widehat{\mathbf{f}}$, K_L	global and local internal loads, linear stiffness
(x_{0a}, y_{0a}), (x_{0b}, y_{0b})	coordinates of nodes a and b in initial configuration

Symbols used in co-rotational shell element

a, $a=1,2,3,4$	node number
T_g	global coordinate system
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$	axes in T_g
T_0, T	co-rotational frames in reference and nominal configurations
$\mathbf{d}_1^0, \mathbf{d}_2^0, \mathbf{d}_3^0$	axes in T_0
$\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$	axes in T
F, R, U	deformation gradient, rotation matrix, tension matrix
$\mathbf{q}$	displacement from reference to current configurations
$\mathbf{q}^n$	relative displacement from reference to nominal configurations
$\mathbf{u}$	relative displacement from nominal to current configurations
$\mathbf{t}_a$	translation part of each node in $\mathbf{q}$
$\boldsymbol{\theta}_a$, $\boldsymbol{\theta}_a^n$, $\boldsymbol{\vartheta}_a$	rotation parts of each node in $\mathbf{q}$, $\mathbf{q}^n$ and $\mathbf{u}$
$\tilde{\boldsymbol{\theta}}_a^n$, $\tilde{\boldsymbol{\vartheta}}_a$	anti-symmetric matrices formed by three components in $\boldsymbol{\theta}_a^n$ and $\boldsymbol{\vartheta}_a$
$\widehat{\mathbf{q}}$	local degrees of freedom
$\widehat{\mathbf{t}}_a$, $\widehat{\boldsymbol{\theta}}_a$	translation and rotation parts of each node in $\widehat{\mathbf{q}}$
$\tilde{\widehat{\boldsymbol{\theta}}}_a$	anti-symmetric matrix formed by three components in $\widehat{\boldsymbol{\theta}}_a$
$\mathbf{r}_a^0$	position vector of each node in reference configuration
I	3×3 identity matrix
R_a	rotation matrix of each node
$\widehat{\mathsf{R}}_a$	local rotation matrix of each node
$\widehat{\boldsymbol{\varepsilon}}$, C	local strain and material matrix

is made that the prebuckling is linear. In reality, this linear assumption for the prebuckling state will often overestimate the buckling load of an important class of engineering problems for which the prebuckling is obviously nonlinear. Cohen [26] and Fitch [27], and later Arbocz and Hol [29,30] have derived the modifications necessary to make Budiansky and Hutchinson's work [28] include prebuckling nonlinearity. Recently, Rahman [13,14,17] and Zagari [11,12] have made use of Arbocz and Hol's [29,30] derivations within a finite element context to consider the nonlinearity of the prebuckling of a structure.

Apart from Koiter reduction methods which are mainly used in the static buckling analysis of structures, the idea of reduced order models has been studied intensively in the past across various disciplines following different approaches. In particular, projection methods based on Krylov subspace algorithms have a tradition due to the numerically efficient and relatively stable generation of orthogonal projection bases [31–33] often with application in structural vibration [20,34,35] and dynamics [16,36–38]. Other disciplines where reduced order models may significantly alleviate the numerical effort spent in repeated solution steps are fluid mechanics [39–42], aeroelasticity [43,44] or optimization [45,46] to mention a few. An exhaustive overview about reduced order modeling techniques and strategies, about fields of application and the analysis of convergence behavior and error-control is provided e.g. in Chinesta et al. [47] or Quaterioni and Rozza [48].

The aforementioned Koiter reduction methods adopt the solution of the reduced order model as a predictor without using any correction step based on the full model [40,41,49–52]. Another family of model reduction techniques combines the prediction stage together with a correction phase [53–55]. These predictor-corrector methods [56–58] are commonly used to trace the nonlinear equilibrium path of structures. Recently, a novel approach, the Koiter–Newton method, has been proposed for the numerical solution of a class of elastic nonlinear structural analysis problems [59–61]. The range of validity of this approach is not limited to the small range near the bifurcation point, since Koiter's asymptotic expansion is applied from the beginning of the equilibrium path rather than only at the bifurcation point. In a series of expansion

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