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Ill-loaded layout optimization of bi-modulus material

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ABSTRACT

In topology optimization of a continuum under multiple loading cases (MLC), if the magnitudes of loads are different obviously, it is hard to obtain a clear component to support the weaker loads. Such MLC are called ill-loading cases (ILC). A new method is presented to solve the layout optimization of a continuum with bi-modulus material under multiple loading conditions (MLC) by using a Q-norm weighting objective which is formed with the Q-norm of weighted structural compliances of MLC. The effects both of the value of Q and bi-modulus behavior of material on the final material distribution are studied. Both of validity and efficiency of the present algorithm are discussed numerically. Results show that the optimal bi-modulus material distribution of a structure under serious ILC can be found if using a small positive value of Q within interval of [0.1, 0.2] and the computational efficiency is very close to that of traditional isotropic material layout optimization.

1. Introduction

Layout/topology optimization is a powerful tool for the conceptual design of a structure/composite material. In topology optimization the layout of material in structure can change, which is the essential difference from the traditional detailed design approaches, e.g., size/shape optimization. However, the computational cost of topology optimization is usually very heavy as comparing with size/shape optimization. Therefore, the popularity of the method in practical design [1] owe to two aspects, i.e., the development of computer technology and related computational methods. The most successful solution methods are as homogenization design based method (HDM) by Bendsøe and Kikuchi [2], solid isotropic material with penalization method (SIMP) by Rozvany et al. [3], evolutionary structural optimization method (ESO) by Xie and Steven [4] and level set method (LST) by Wang et al. [5].

In practical engineering, such materials as concrete, cast iron, plastic and rubber, are used popularly. Mechanical experiments show that the tensile and compressive moduli of the materials are different under linear elastic deformation along the same direction. In general, the materials are called bi-modulus materials. The mechanical property of a bi-modulus material is stress dependent which results in many times of structural reanalysis for obtaining the accurate deformation of a structure with such material [6]. To avoid the behavior of stress-dependent of a bi-modulus material, approximation schemes are presented [7–10]. In such approximation, the original piecewise linear constitutive curve is approximated with a continuous differentiable curve. On the other hand, material replacement methods are also suggested [11–16] in layout optimization of a continuum as the different behavior in tension and compression exists.

In practical engineering, a structure is generally subjected to multiple loading cases (MLC) [17–22]. In the work by Sui et al. [19], they presented a multiple-level weighting scheme to deal with the topology optimization under ill-loading cases. In their work, the loading cases were divided into two groups according to the magnitudes of the loads. Topology optimization of structure under the higher loads was firstly carried out. And the second level of topology optimization of structure under lower loads was implemented on the results of previous level of topology optimization. In 2006, they [20] proposed a new method to select the weight functions of loads by considering strain energy constraints in optimization.

The approach to deal with ILC design is still complicated. Besides, the stiffness design of a continuum under MLC discussed in above works does not consider material nonlinear, e.g., bi-modulus material in structure. That is the motivation of our present work. Simultaneously, in the present work, the power exponent weighting scheme of objective functioin is adopted. The value of the power exponent is in the interval of (0, 1) to deal with ill-loading cases. Further, the effects of both the value of the power exponent and bi-modulus behavior of material on the optimal topologies are to be discussed numerically.

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2. Methodology

Essentially, finite element method (FEM) is adopted to give structural deformation analysis is this work. Material replacement method [16] is adopted to solve bi-modulus stiffness design. And Q-norm weighting scheme is presented to solve topology optimization of a continuum under MLC.

2.1. Formulation of weighted optimal stiffness design

In the present work, only linear elastic structure is considered. The optimal stiffness design of a structure with bi-modulus material under weighted MLC is defined as

$$\begin{aligned} &\text{Find} \quad \{\rho_m | m \in \Omega\} \\ &\min \quad c_w = \left[\sum_{i=1}^{N_{LC}} (w_i \overline{c_i})^Q\right]^{1/Q} \\ &\text{s.t.} \quad \sum_m v_m \rho_m - f_v \cdot V_0 = 0 \\ &\quad K_i \cdot U_i = P_i, \quad (i = 1, 2, \cdots, N_{LC}) \\ &\quad \sum_{i=1}^{N_{LC}} w_i = 1.0 \\ &\quad \rho_m \in [\rho_{\min}, \quad 1.0], \quad (m \in \Omega) \end{aligned} \tag{1}$$

where ρ_m is the relative density of the m-th element in design domain, i.e., Ω. c_w denotes the weighted structural compliance under N_{LC} types of loading conditions. w_i is the linear weighting coefficient. $Q \in (0, \infty)$ is power exponent for weighting scheme which is different from that given by Luo et al. [21]. In their work, $Q=2$ was used and common MLC problem was considered. $\overline{c_i}$ is the modified structural compliance under the i-th loading condition. K_I is the global stiffness matrix of structure under the i-th loading condition, U_j and P_j are the global nodal displacement vector and nodal force vector, respectively. ρ_{min} is set to be 0.001 in this work to avoid singularity of the global stiffness matrix of structure with fixed mesh scheme.

In optimization, the relative densities of elements in design domain are design variables. In order to use mathematical programming approach to solve the optimization, the binary design (0/1) problem is relaxed to be a continuous design variable problem, i.e., $\rho_m \in [\rho_{min}, 1.0]$. To reduce the amount of the mid-density elements, the stiffness matrix of porous material with the relative density of ρ_m is given according to the power-law rule [23], i.e.,

$$D_{m,\rho} = \rho_m^p \cdot D_{m,0} \tag{2}$$

where $D_{m,0}$ is the stiffness matrix of the m-the element without pores.

2.2. Material replacement scheme

Fig. 1 gives the elastic constitutive curve of a bi-modulus material. The elasticity of a bi-modulus material is stress-dependent. Take $E_T = \tan\alpha$ and $E_C = \tan\beta$ as the tensile modulus and compressive modulus of bi-modulus material, respectively. And define the ratio between E_T and E_C using

$$R_{TCE} = E_T / E_C \tag{3}$$

Fig. 2 shows that the bi-modulus material shows isotropic under pure tension (Fig. 2a) or under pure compression (Fig. 2b). Only when under complex stress state (Fig. 2c), the elasticity of bi-modulus material shows orthotropic and the principal directions of material is in accordance with the principal directions of stress tensor. For a complicated deformation of a structure with bi-modulus material, generally, it has components or areas under complex stress states. So, traditionally, many times of structural reanalysis is required for obtaining the enough accurate results of a structure with bi-modulus

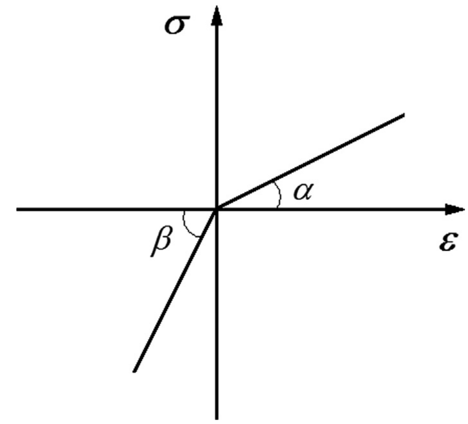

Fig. 1. Stress(σ)-strain(ε) curve of a bi-modulus material ($\alpha \neq \beta$).

material [6,7]. In reanalysis, the elastic matrix and the principal directions of material varies alternatively.

Using traditional method to solve topology optimization of a continuum with bi-modulus material, there exist at least two levels of iterations, i.e., the inner iteration for finding out the accurate deformation of structure and outer iteration for updating the design variables. It is known, in the first several iterations for optimization, the results are not optimal. So, our suggestion is to divide the inner iteration and merge them into the outer iteration. In other words, there is only one time of structural analysis for each update of design variables. Simultaneously, the local stiffness should be adjusted with respect to the local stress state in the previous step.

However, the elasticity shows orthotropic with unset material principal directions if the bi-modulus material is under complex stress states. Theoretically, the number of orthotropic materials predefined in simulation should be equal to that of finite elements in structure. It is incredible for a complicated structure with great quantity of elements. Therefore, the feasible way to overcome this difficulty is to define several stress-independent materials to replace the original bi-modulus material. Further, the easiest way is to use two isotropic materials to replace the original material. From above, E_T and E_C can be selected as the moduli of the two isotropic materials. Clearly, the difference between the mechanical behaviors of structure with original material and with new isotropic materials is mainly caused by the material replacement happening in the elements which are under complicated stress states.

2.3. Modification factor of local stiffness

In simulation, the original bi-modulus material is replaced with isotropic materials in structure. To obtain the same loading-deformation relations of structure, the local stiffness must be the same for the structure either with original bi-modulus material or with the new replacement material (isotropic material). Under the same complex stress state, the (original) element with bi-modulus material shows different deformation from that (the new replaced element) with isotropic material. And their strain energy must be different, too. This conclusion is obtained based on the same amount of material in both elements. Fortunately, the amount of material in optimization is changeable. The local stiffness (e.g., strain energy) can be forced to be identical for two elements by modifying the amount of material in element. Therefore, the modification of local stiffness is, actually, a further update of material amount in element. The modification factor of the local stiffness can be calculated by the following equation:

$$f_m = \max\left[10^{-6}, (\text{SED}_m^{effective} / \max(10^{-30}, \text{SED}_m))\right] \tag{4}$$

where the total SED of the m-th element is

$$\text{SED}_m = \text{TSED}_m + \text{CSED}_m \tag{5}$$