Finite Elements in Analysis and Design 92 (2014) 12–25

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Mixed-dimensional finite element coupling for structural multi-scale simulation

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ARTICLE INFO

Article history:
Received 2 April 2014
Received in revised form
30 June 2014
Accepted 12 July 2014
Available online 24 August 2014

Keywords:
Finite elements
Mixed-dimensional coupling
Multi-scale simulation
Constraint equations
Linear and nonlinear analysis

ABSTRACT

Multi-scale finite element (FE) modeling and analysis of engineering structures have become very necessary in order to provide both global and local structural information for a comprehensive assessment of structural safety. The multi-scale FE modeling needs FE coupling methods to combine mixed-dimensional finite elements, such as beam-to-shell and plate-to-solid, in a single structural model. This paper presents a new mixed-dimensional FE coupling method that can achieve both displacement compatibility and stress equilibrium at the interface between the different element types. The principle of virtual work is first used to derive both linear force and displacement constraint equations for the interface. A numerical method compatible with commercial FE codes is developed to figure out the linear constraint equations which satisfy both displacement compatibility and stress equilibrium conditions at the interface. The proposed coupling method is then extended to nonlinear mixed-dimensional FE coupling problems. Finally, the proposed coupling method is applied to a number of test cases including linear beam-to-plate, beam-to-shell and beam-to-solid interface problems, a linear frame structure, and a beam-to-shell buckling problem. The obtained results are also compared with those from the existing methods. It reveals that the proposed coupling method can handle both linear and nonlinear mixed-dimensional FE coupling problems more accurately than the existing methods and that it can be applied to a structure satisfactorily for multi-scale simulation.

1. Introduction

Finite element (FE) modeling and analysis of an engineering structure nowadays are a common practice to understand structural behavior and assess structural safety. The FE modeling using small-scale elements can usually improve the accuracy of numerical simulation of a structure but it can also lead to a huge computation cost or a difficulty to run. The FE modeling using relatively large-scale elements may capture global structural behavior but it may not be able to detect local structural feature. The multi-scale FE simulation can provide a better solution in this aspect [1–4]. By taking a frame structure shown in Fig. 1 as an example, the local joints are simulated with shell elements of small scale while the other components in the frame are simulated with beam elements of large scale. Such a multi-scale simulation can capture not only the global structural behavior in terms of displacement and acceleration but also the local joint behavior in terms of stress and strain without a huge computation cost.

Since different types of elements (beam, plate, shell, and solid) have different number of degrees-of-freedom (DOFs), the multi-scale FE simulation needs a rational FE coupling method to combine mixed-dimensional finite elements at their interfaces into a single structural model. The challenging issue in the multi-scale FE modeling is therefore how to guarantee the rationality of the coupling method so that it can achieve both displacement continuity and stress equilibrium in the region around the interface between the different types of elements.

Broadly speaking, there are two major coupling methods currently available: volume coupling and surface coupling [5]. Volume coupling refers to a region in which different models co-exist and it is usually realized using the Arlequin method [6]. The Arlequin method is best suited for coupling different physical models such as continuum particles [7,8] among others. In surfacing coupling, there is no overlapping of different models and different models can be coupled using one of the following methods: (a) Lagrange multiplier method; (b) penalty method; (c) transition element method; and (d) multipoint

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http://dx.doi.org/10.1016/j.finel.2014.07.009
0168-874X/

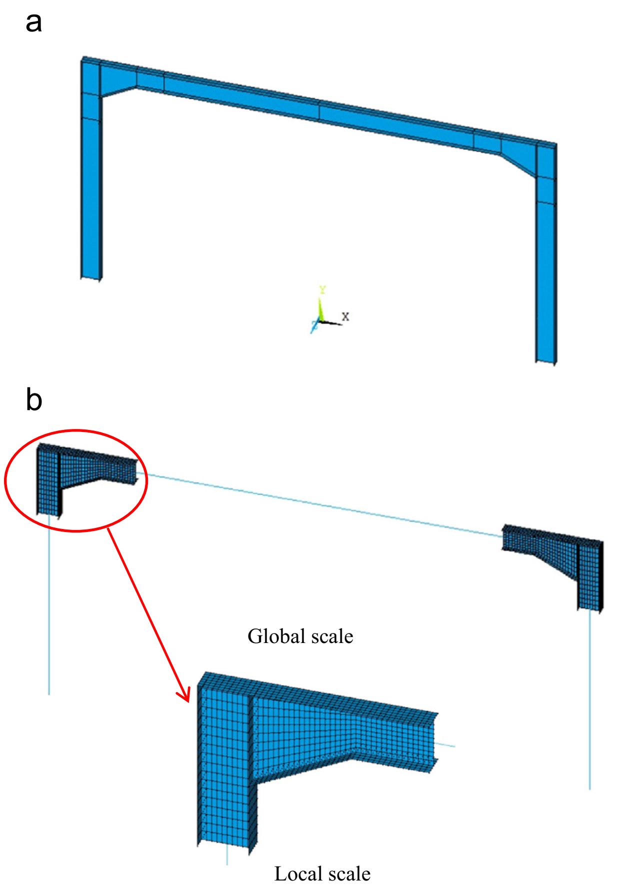

Fig. 1. Multi-scale modeling of a frame structure: (a) frame structure and (b) multi-scale model.

constraint (MPC) method. Both the Lagrange multiplier method and the penalty method have their own disadvantages: in the former it is the introduction of extra unknowns and in the latter it is the choice of the penalty parameter. Transition elements employing either reduced or full integration can be used for shell–solid transition [9–12], beam–solid transition [13,14], and beam–shell transition [15,16]. Unfortunately, the transition elements have not been widely adopted because of its limitations. Transition elements can only be used to a one-to-one coupling of elements, and different element transitions require different formulations which make it difficult and impractical for a commercial FE code.

The MPC method is an attractive coupling method for coupling mixed-dimensional elements by using constraint equations for nodal displacements at the interface. The MPC method can be used for static and dynamic analysis of linear or nonlinear structures [17–22]. This method is easy to access in a few commercial FE codes. For instance, the RBE3 MPC method provided in ANSYS code [20] can automatically establish constraint equations for coupling the different types of elements. There are mainly two types of the MPC method: the rigid interface method and the deformable interface method. The rigid interface method uses rigid beams to connect nodes of different types of elements, such as CERIG in ANSYS and MPC-BEAM in ABAQUS [20,21]. The rigid interface method, however, yields stress disturbance at the interface because the interface is defined as a rigid interface. The deformable interface method uses a concept of force distribution at the interface, such as RBE3 in ANSYS and the distributing coupling method in ABAQUS. The deformable interface method allows the interface deformation with stress distributions at the interface. RBE3 in ANSYS allows the motion of the master node equal to the average of the slave nodes in which only translational DOFs of the slave nodes involves in the constraint equations. The force and moment are distributed to the slave nodes by weighting factors and the distance from the center of slave nodes times weighting factors, respectively. The distributing coupling method in ABAQUS constrains the motions of the coupling nodes to the motion of a reference node in an average sense. Forces and moments at the reference node are distributed either as a coupling node-force only or as a coupling node-force and moment. Both RBE3 and distributing coupling methods have the sense of force and moment distribution by means of weighting factors but the accuracy of stress distribution at the interface resulting from force and moment distribution by means of weighting factors is questionable. On the other hand, another deformable MPC method was proposed based on the direct assumption of stress distribution at the interface and the equal work done by the stresses and forces at the interface [18]. As for the scheme of implementing the MPC method into the finite element software, there are three common types: penalty scheme; Lagrange multiplier scheme and elimination scheme [23–25]. The penalty scheme only enforces the MPC method in an approximate manner: the choice of penalty coefficient is a dilemma and it can be neither small nor large. The Lagrange multiplier scheme increases the number of DOFs in the system and destroys the banded and positive definite nature of the stiffness matrix. The elimination scheme enforces the MPC method by eliminating dependent DOFs. It needs more matrix operations and destroys the symmetry and bandwidth of the original stiffness matrix, and the choice of independent DOFs is non-unique. Although the deformable MPC method seems more rational and effective, inaccurate constraint equations due to inappropriate stress distribution assumptions may result in stress disturbance at the interface.

This paper presents a new deformable MPC coupling method that can achieve both displacement compatibility and stress equilibrium at the interface between the different element types. The principle of virtual work is first used to derive both linear force and displacement constraint equations for the interface. A numerical method compatible with commercial FE codes is developed to figure out the linear constraint equations which satisfy both displacement compatibility and stress equilibrium conditions at the interface. The proposed coupling method is then extended to nonlinear mixed-dimensional FE coupling problems. Compared with the McCune's method, the proposed method drops the assumption that beam/plate/shell theory governs the stress distribution at the interface between elements. To validate the proposed coupling method, a number of finite element test cases are finally examined. These cases include linear beam-to-plate, beam-to-shell and beam-to-solid connections for linear mixed-dimensional FE coupling, a linear frame structure for structural multi-scale simulation, and a beam-to-shell buckling problem for nonlinear mixed-dimensional FE coupling. The computation results are also compared with those from the existing methods to demonstrate accuracy and robustness of the new algorithm. While the method proposed in this paper is suitable for the mixed-dimensional FE coupling of beam and plate, beam and shell, and beam and solid in either linear or nonlinear multi-scale simulation of frame structures, there is a possibility of extending the proposed method to the mixed-dimensional FE coupling of shell and solid, plate and solid, and other, but it needs further study.

2. Linear constraint equations

For static linear structural analysis, the interface coupling for elements of different DOFs can be established using linear constraint equations. An example of interface coupling for a two-dimensional beam and plate connection, as shown in Fig. 2, is used to illustrate how the constraint equations can achieve both displacement compatibility