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Structural topology optimization based on system condensation

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ABSTRACT

Topology optimization problem requires repeated evaluations of the objective function and design sensitivity in the design domain with various density distributions. A repeated computations in the optimization process, requires a large amount of computing time and resources. These issues have inspired the development of optimization techniques combined with a system reduction. In order to reduce the system, this study employs a system dynamic condensation method based on selected primary degrees of freedom. Based on a system reduction, this study performs a topology optimization to maximize the eigenvalue and linear summation of each eigenvalue. In the optimization procedure, mode tracking method, called MAC, is used to pursue target modes, and the design sensitivity is calculated by a method of the rigid body mode separation assuring the reliability of sensitivity regardless of the design variable perturbation size. Each result of the numerical examples based on the reduction system is compared to that of the full system. Through a few numerical examples, it is demonstrated that the proposed method can provide efficient and reliable results in topology optimization.

1. Introduction

Traditionally, the size and shape optimization has been the main stream in the structural optimization field. Besides these two kinds of optimization fields, topology optimization came into the limelight within decades. The definition of topology in mathematics stands for the invariant of spatial one-to-one mapping, which is continuous and includes the inverse mapping. However, size optimization, which treats the coefficient of a cross section, has a disadvantage, which is optimized within a given shape. Shape optimization, which deals with the modification of external shape, has a disadvantage in that the distorted mesh should be regenerated when elements are distorted during the optimization process. However, topology optimization has nothing to do with these disadvantages but is applicable for design without a basic layout.

Topology optimization originated as layout optimization by Rozvany and Prager in the 1960s. After this outstanding work, Bendsøe and Kikuchi et al. developed topology optimization to minimize compliance in static problems [1–3]. Then, topology optimization for maximizing eigen-frequency has been studied by Ma and Kikuchi [4,5]. Even though many applications of topology optimization have been studied regarding to the static problem, topology optimization of dynamic problem has been limited for some reasons. The change of mode sequence during optimization of eigen-frequency makes the cost function non-smooth. Moreover, the sensitivity analysis of the cost function could be discontinuous and the optimization process of a vibration problem could be non-convex. The main root of these problems is related to the mode tracking conditions during the optimization process of eigen-frequency. If the optimization does not pursue reliable mode tracking, a proper sensitivity analysis is not guaranteed so that the improper mode tracking becomes the main obstacle in optimization. Then, it is required to pursue the correct target mode during the process of the topology optimization. Eldred et al. have proposed two kinds of the mode tracking methods for eigenvalue problem [6]. One is the higher order eigen-pair perturbation algorithm and the other is the cross-orthogonality check method. Even if the higher order eigen-pair perturbation algorithm guarantees quite accurate mode, it is not conformable to use the intermediate process of topology optimization [7]. In this paper, the cross-orthogonality check method, called modal assurance criterion (MAC), is used to track target eigenmode. MAC verifies the correlation between experimental and numerical mode shape [8]. For the application of MAC to the presented mode tracking in topology optimization, we aim at a standard eigenmode shape as a target mode shape at initial configuration. And, we compute the MAC value of the candidate eigenmodes of the updated structure configure to track the target mode shape in each iteration.

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The optimization process of eigen-frequency needs many repeated analyses in dynamic problems. In particular, the analysis of a large scale structure takes enormous computing time. To make up for this trouble, this study employs the reduction system, which is efficient in terms of calculating time and reliable in terms of accuracy. We propose for the first time a methodology of topology optimization combined with the dynamic condensation system. The reduction system has been applied to various research fields, such as eigenvalue analysis, sub-structuring schemes and structural optimization based on sub-domain [9–11]. The reduction system has been developed to improve the problems that consume excessive computing time and computer resources in large-scale analysis. There are two kinds of methods based on the primary degrees of freedom (PDOFs) and the mode-based structural reduced order model (ROM) in the reduction system. We use the reduction method, so called, condensation method, based on PDOFs which we selected by a two-level condensation scheme (TLCS) [9].

The key issue in this study is how to perform the structural topology optimization by combining it with the reduction system. This study proposes an efficient topology optimization based on the system reduction. Section 2 briefly introduces the construction of the reduction system based on the PDOFs. Section 3 presents the general topology optimization including the mode tracking method. Section 4 evaluates the formulation of the sensitivity in topology optimization. Section 5 presents the maximization of the eigen-frequencies of a beam and plate by the proposed optimization method. Through numerical examples, the efficiency and reliability of the proposed method are demonstrated when the reduced system is applied to the topology optimization. The accuracy and efficiency of the reduced system approach for each example is compared to that of full system.

2. Review of system reduction method

In the process of the topology optimization, the eigenvalue and its design sensitivity is obtained by the reduction system based on PDOFs. Because the accuracy of the reduction system is highly dependent on the choice of PDOFs, the TLCS, which was verified by previous research, was used for the proper selection of the PDOFs. This scheme consists of two steps, the selection of the candidate area and PDOFs. In the first step, the scheme selects the candidate area by Rayleigh energy ratio. In the second step, the sequential elimination method (SEM) [12] is applied to determine the final PDOFs. Fig. 1 is the overall schematic of the TLCS [9]. Fig. 1(a) is the model configuration for analysis and Fig. 1(b) shows the selected candidate area, while Fig. 1(c) shows the PDOFs selected by the SEM. This method, which is based on the Improved Reduced System (IRS, [13]) was proposed for an efficient reduction system by Kim and Cho [9]. Section 2.1 and Section 2.2 briefly introduce the formulation of the two steps for constructing the reduction system. In the first step, in which the candidate regions for PDOFs are selected, the modified Rayleigh quotient considering the density of elements is evaluated to consider the void element in topology optimization.

2.1. The first stage for selecting the candidate area

2.1.1. Process 1: Extraction of Ritz vectors

To obtain the Ritz vectors, an initial static vector $\{\mathbf{x}_1^*\}$ is calculated by static analysis as Eq. (1a). The first Ritz vector $\{\mathbf{z}_1\}$ is obtained by the mass-normalization procedure of this static vector as shown in Eq. (1b).

$$\begin{cases} \{\mathbf{x}_1^*\} = [\mathbf{K}]^{-1}\{\mathbf{F}_1\} & (a) \\ \{\mathbf{z}_1\} = \{\mathbf{x}_1^*\} / \sqrt{\{\mathbf{x}_1^*\}^T[\mathbf{M}]\{\mathbf{x}_1^*\}} & (b) \end{cases} \tag{1}$$

where the entity of $\{\mathbf{F}_1\}$ is the diagonal term of the mass matrix.

The first Ritz vector which obtained from Eq. (1b) is used to find the next static vector in Eq. (2a). The ith static vector $\{\mathbf{x}_i^*\}$ which calculated in Eq. (2a) becomes Mass-orthogonal to the previously obtained Ritz vectors by the Gram–Schmidt procedure of Eq. (2b). Then, ith Ritz vector $\{\mathbf{z}_i\}$ is obtained by the mass-normalization procedure of Eq. (2c).

$$\begin{cases} [\mathbf{K}]\{\mathbf{x}_i^*\} = \{\mathbf{F}_i\}, \{\mathbf{F}_i\} = [\mathbf{M}]\{\mathbf{z}_{i-1}\} \quad (i>1) & (a) \\ \{\mathbf{x}_i^*\} = \{\mathbf{x}_i^*\} - \sum_{k=1}^{i-1} [\{\mathbf{x}_i^*\}^T[\mathbf{M}]\{\mathbf{z}_k\}]\{\mathbf{z}_k\}^T \quad (i>1) & (b) \\ \{\mathbf{z}_i\} = \{\mathbf{x}_i^*\} / \sqrt{\{\mathbf{x}_i^*\}^T[\mathbf{M}]\{\mathbf{x}_i^*\}} & (c) \end{cases} \tag{2}$$

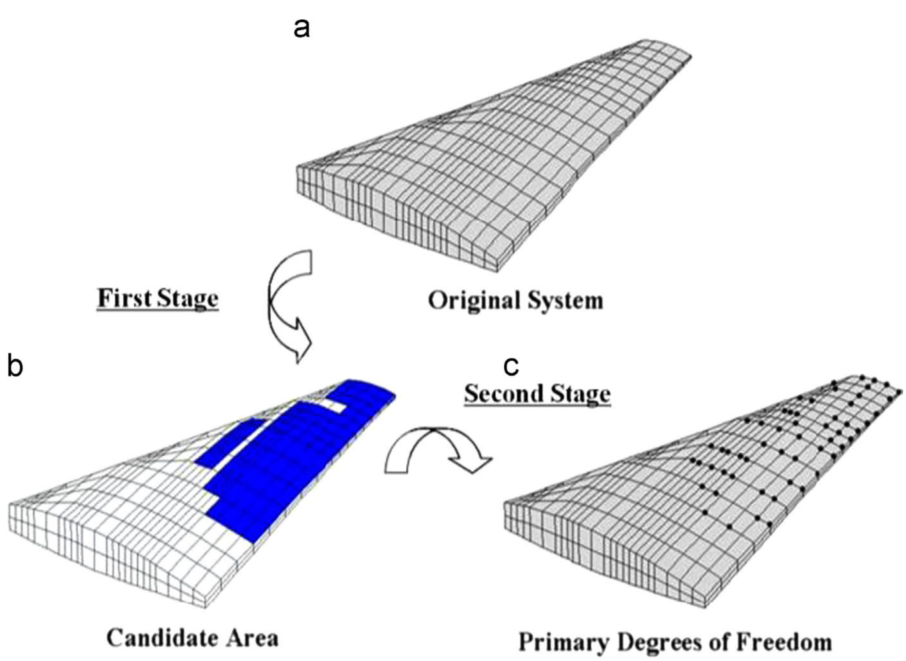

Fig. 1. Schematic of two-level condensation scheme (TLCS).