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Variationally consistent quadratic finite element contact formulations for finite deformation contact problems on rough surfaces

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ABSTRACT

Although different discrete formulations for contact problems have been widely studied during the last decade, the numerical simulation of complex industrial applications is still challenging. While suitable Lagrange multiplier based formulations are well-known for their consistency and stability in the case of classical model problems of Coulomb type, rough surface contact laws and additional multi-point constraints are much less understood. In this paper, we focus on a quadratic finite element approach for quasi-static calculations and extend ideas from our previous work on constitutive contact laws combined with suitable solutions for multi-point constraints like cyclic symmetry on the contact boundary. The popular dual mortar method is used to enforce the contact constraints in a variationally consistent way without increasing the algebraic system size. To avoid possible consistency errors of the dual mortar approach in case of large curvatures or gradients in the contact zone, an alternative quadratic Petrov–Galerkin mortar formulation is presented. Numerical examples demonstrate the robustness of the derived numerical algorithm. Special focus is set to industrial motivated applications involving large deformations and plastic effects as well as rough surfaces on the micro-scale.

1. Introduction

Although a lot of progress has been made in the implementation of contact algorithms in commercial codes, solving non-linear contact problems within the FEM framework is still a challenging task when facing large plastic deformations, non-linear material laws and multi-constrained contact problems of industrial relevance. This led to new research activities in this field. For a more general overview on contact problems, we refer to the monographs by Johnson [1] or Bowden/Tabor [2] for contact between rough surfaces, the monographs by Kikuchi/Oden [3] and Eck [4] for existence and uniqueness results, to the monographs by Willner [5], Laursen [6] and Wriggers [7] for computational aspects and to the monograph by Cotrell/Hughes/Bazilevs [8] for isogemetrical formulations.

The most popular approaches to enforce the contact conditions are the penalty method and the Lagrange multiplier method or combinations of both like the augmented Lagrange method or the perturbed Lagrange method. Within the penalty framework a spring between a slave location and a master surface is generated modelling the contact traction as a function of the displacements, if the non-penetration condition is violated. This approach is often combined with a node-to-surface formulation [9]. More recent contact formulations combine the penalty method with a Gauss point-to-surface formulation [10,11] to overcome the drawbacks of the node-to-surface formulation.

If contact problems between two deformable bodies and non-matching meshes are considered, the Lagrange multiplier setting is strongly related to mortar methods introduced originally in the context of domain decomposition techniques by Bernardi [12] and Ben Belgacem [13]. Mortar methods are characterized by the introduction of an additional variable to model the traction across the interface and a weak formulation of the interface constraints leading to a surface-to-surface approach. They were adapted to small deformation contact problems by Ben Belgacem [14] and later expanded to large deformation contact [15], curved interfaces and large sliding [16,17] and dynamical problems [18–20].

Among the mortar methods, the dual mortar method [21] has become of interest [22–24] since it enforces the non-penetration condition in a variationally consistent way without increasing the system size. Exploiting the duality between displacement and traction spaces in the definition of the discrete nodal basis functions, the Lagrange multiplier can be locally condensed from the global system before solving. Thus a pure displacement based system results in each load-step. Combined with a semi-smooth Newton method [25,26], one obtains a flexible, robust and efficient

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tool for contact problems since all non-linearities of the system (material, geometry and contact conditions) can be handled within the same iteration loop.

Another current area of research is the isogeometric analysis for contact problems to enable a tighter connection between CAD and FEA. Here the same smooth and higher order basis functions are used for the representation of the CAD geometry as well as for the FEA solution fields. Although showing very promising results in the field of domain decomposition [27] or in contact simulation combined with a Gauss point-to-surface [28–30] or a mortar formulation [30–34], the isogeometric methods complicate the local problem treatment due to the broader support. This possibly results in inaccuracies in the boundary between contact and no-contact regions [29] and can be seen as a limitation to industrial applications. Therefore this paper uses the well studied dual mortar method in combination with quadratic finite elements as a reasonable compromise between linear finite element formulations and the full isogeometrical approach.

Most publications dealing with the mortar method restrict their considerations to linear finite elements. But for industrial applications, these possibly show numerical artefacts like shear locking, volumetric locking and hour-glassing. Quadratic finite elements on the other hand approximate curvilinear interfaces more accurately and lack these numerical artefacts while at the same time avoiding the costly step switching to the isogeometrical approach. Thus they are quite attractive from the point of accuracy and computational complexity. Quadratic finite element formulations have been studied for the standard mortar formulation by Puso et. al. [16] and for the dual mortar formulation by Popp et. al. [35] expanding the ideas from Lamichhane and Wohlmuth [36]. Here these ideas are picked up and combined with the previous work of the authors on constitutive contact laws due to rough surfaces on the micro-scale [37,38].

Another critical point dealing with the dual mortar method can arise in set-ups with curved interfaces leading to a possibly non-physical evaluation of the weighted dual gap due to the non-positive part of the dual basis functions. A similar issue has also been regarded in case of mesh-tying and simple contact situations in [39]. In case of general contact problems, the problem with the dual gap can be avoided using a Petrov–Galerkin dual mortar formulation combining the benefits of the standard and the dual Lagrange multiplier method in the linear setting [40]. Here this idea is picked up and extended to quadratic finite elements with microscopic rough surfaces.

The third criteria bridging the gap to industrial applications of the dual mortar method is the ability to handle hanging nodes on the slave surface or single and multi-point constraints like directional blocking and cyclic symmetry in the contact zone. For the latter case, this results in an over-constrained system for these slave nodes carrying both, the Lagrange multiplier and a multi-point constraint. Here we extend an idea presented in [41] for mesh tying. This also solves the problem of hanging nodes on the slave surface.

The outline of the present work is as follows: We start with a brief problem definition in Section 2. The extension of the dual mortar method with regularized contact conditions to quadratic elements is presented in Section 3. Two different approaches are studied: a quadratic–quadratic method and a quadratic–linear method. In Section 4, the quadratic–linear dual mortar formulation is combined with a Petrov–Galerkin approach to overcome the problem with the dual gap. The adoption of the algorithm solving over-constraints or hanging nodes is presented in Section 5. In Section 6, various numerical examples demonstrate the robustness of the derived algorithms. The examples involve plastic effects as well as rough surfaces on the micro-scale. Special focus is set here to industrially motivated examples: a film forming process, a fir tree contact in an abstract turbine and a complex disk blade foot contact with a retainer.

2. Problem definition

We start with a brief overview on the quasi-static 3*D* two-body contact problem in a non-linear elasticity setting [3,5]. As one can see in Fig. 1, the two contacting bodies, a slave body s and a master body m with domains $\Omega^\alpha \in \mathbb{R}^3$, $\alpha = s, m$, are undergoing motion during the time interval $[0,T]$ described by the mapping $\boldsymbol{\varphi}^\alpha : \Omega^\alpha \times [0,T] \to \mathbb{R}^3$, $\boldsymbol{X}^\alpha \mapsto \boldsymbol{x}^\alpha = \boldsymbol{\varphi}^\alpha(\boldsymbol{X}^\alpha, t)$, $\alpha = s, m$, which maps material points $\boldsymbol{X}^\alpha$ of the reference configuration to the current configuration $\boldsymbol{x}^\alpha$. The displacements are defined as $\boldsymbol{u}(\boldsymbol{X},t) = \boldsymbol{\varphi}(\boldsymbol{X},t) - \boldsymbol{X}$ with $\boldsymbol{u} = (\boldsymbol{u}^s, \boldsymbol{u}^m)$ on $\Omega = \Omega^s \times \Omega^m$. The boundary of the bodies $\partial\Omega^\alpha$ is divided into three disjoint sets, the Dirichlet boundary Γ_D^α, the Neumann boundary Γ_N^α and the potential contact boundary Γ_c^α with their spatial counterparts γ_D^α, γ_N^α and γ_c^α. It holds $\overline{\Gamma}_D^\alpha \cup \overline{\Gamma}_N^\alpha \cup \overline{\Gamma}_c^\alpha = \partial\Omega^\alpha$. In terms of the displacements $\boldsymbol{u}$, the deformation gradient $\boldsymbol{F} = \nabla[\boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X},t)]$, the right Cauchy–Green tensor $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$ and the Green–Lagrange strain tensor $\boldsymbol{E} = 1/2(\boldsymbol{C} - 1) = 1/2(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T + \nabla\boldsymbol{u}\nabla\boldsymbol{u}^T)$ can be defined. Non-linear material laws are considered assuming the existence of an energy density $W = W(\boldsymbol{C})$ including isotropic plasticity with multiplicative split [42–44].

The strong form of the two-body contact problem. Find $\boldsymbol{u}^\alpha : \Omega^\alpha \times [0,T] \to \mathbb{R}^3$ satisfying

$$\text{balance equation} \quad \left\{ \nabla \cdot \boldsymbol{P}^\alpha + \rho_0^\alpha \overline{\boldsymbol{b}}^\alpha = 0, \quad in \quad (0,T] \times \Omega^\alpha, \right. \tag{1a}$$

$$\text{boundary conditions} \quad \begin{cases} \boldsymbol{u}^\alpha = \overline{\boldsymbol{u}}^\alpha, & in \quad (0,T] \times \Gamma_D^\alpha, \\ \boldsymbol{P}^\alpha\boldsymbol{n} = \overline{\boldsymbol{t}}^\alpha, & in \quad (0,T] \times \Gamma_N^\alpha, \\ \boldsymbol{P}^\alpha\boldsymbol{n} = \boldsymbol{t}_c^\alpha, & in \quad (0,T] \times \Gamma_c^\alpha, \end{cases} \tag{1b}$$

$$\text{initial conditions} \quad \left\{ \boldsymbol{u}^\alpha(\cdot, 0) = \boldsymbol{u}_0^\alpha, \quad \text{in} \quad \Omega^\alpha, \right. \tag{1c}$$

where $\boldsymbol{P}$ is the first Piola–Kirchhoff stress tensor. The first equation describes the balance of momentum with $\nabla \cdot \boldsymbol{P}^\alpha$ being the internal forces and $\rho_0^\alpha \overline{\boldsymbol{b}}^\alpha$ being the body forces acting on Ω^α. On the Dirichlet boundary, we assume $\overline{\boldsymbol{u}}^\alpha = 0$ and on the Neumann boundary $\boldsymbol{P}^\alpha\boldsymbol{n} = \overline{\boldsymbol{t}}^\alpha$, where $\overline{\boldsymbol{t}}^\alpha$ is a given surface force. The contact traction $\boldsymbol{t}_c^\alpha$ and the actual contact zone are not known a priori and have to be determined numerically. For the quasi-static approximation used in this paper, the inertia forces are neglected and the time interval $[0,T]$ is split into small load increments $t_0 = 0, \ldots t_n = T$.

For the contact constraints, we define for each point on the slave surface the orthogonal system $(\boldsymbol{n}, \boldsymbol{\tau}_1, \boldsymbol{\tau}_2)$, with $\boldsymbol{n} = \boldsymbol{n}(\boldsymbol{X},t)$ being the current normal on the slave surface in the point $\boldsymbol{X}$ at time t and $\boldsymbol{\tau} = (\boldsymbol{\tau}_1, \boldsymbol{\tau}_2)$ the corresponding tangents, see Fig. 1. In terms of this nomenclature, we define the projection operator $P_t : \Gamma_c^s \to \Gamma_c^m, \boldsymbol{X} \longmapsto P_t(\boldsymbol{X})$ which takes a point $\boldsymbol{X}$ of the slave surface in the current configuration and projects it onto the master surface in direction of the current normal $\boldsymbol{n}(\boldsymbol{X},t)$, see Fig. 1. With the projection operator P_t, the gap function in the normal direction can be defined for the current configuration $G_n(\boldsymbol{X}^s,t) := [\boldsymbol{X} + \boldsymbol{u}(\boldsymbol{X},t)]_n = \boldsymbol{n}^T \cdot (\boldsymbol{\varphi}^s(\boldsymbol{X}^s,t) - \boldsymbol{\varphi}^m(P_t(\boldsymbol{X}^s),t))$. Finally the Lagrange multiplier is defined as the negative contact traction on the slave side $\boldsymbol{\lambda} = -\boldsymbol{t}_c^s$.

Now the normal contact conditions with regularization due to the roughness on the micro scale on $\Gamma_c{}^s$ can be stated as follows, see Fig. 2(a) and (b):

$$[\boldsymbol{u}]_n - G_n^c(\lambda_n) \le g_0 \wedge \lambda_n \ge 0 \wedge \lambda_n \cdot ([\boldsymbol{u}]_n - G_n^c(\lambda_n)) = 0.$$

Here the linearised version of the normal contact is utilized assuming small deformations on the contact boundary with