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A numerical method for eigensolution of locally modified systems based on the inverse power method

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ABSTRACT

Rapid reanalysis of eigensolutions after modification is a problem of considerable practical importance. Several methods have been developed for computing eigenvalues by using modified parts. This paper proposes just such a method based on inverse power method, using the DOF of only the modified part. This approach enables exact solutions for eigenvalues to be found quickly, regardless of the magnitude of the modification, using the degree of the condensed modified part. The advantages of the proposed method are examined comparing the solutions of inverse power method by several numerical examples. This approach will be useful for a modified system with large degrees of freedom and small modification part.

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1. Introduction

Vibration engineering encompasses a wide variety of fields, from mechanical vibrations to electrical oscillations and the swaying of large structures. Dynamic analysis of eigenvalues and eigenvectors during the free vibration of such systems (kinetics) is a fundamental branch of engineering, and a large number of numerical methods have been proposed for efficiently performing such analyses. Recent advances in the finite element method (FEM) and in large-scale computing have enabled analysts to treat many more degrees of freedom (DOF), and increasingly versatile software is being developed to accommodate these factors. However, if any parameter is changed (e.g., shape, material, initial conditions or environmental conditions) in a previously analyzed system, the entire system has to be reanalyzed. Eigenvalue analysis is a much more arduous task than the corresponding static analysis, and the computation time for the former is generally several times or even several tens of times greater than that for the latter. This can make the process quite expensive if a system undergoes multiple revisions, since analyzing the system after the revision requires just as much time, labor and money as analyzing the original system.

"Reanalysis" is the term used to describe the partial analysis (as opposed to a full analysis) that is performed in order to obtain data efficiently when a portion of a matrix has been altered due to a design revision or some other change. If a convenient method can be found for performing reanalysis that utilizes the previously calculated eigenvalues and eigenvectors, it will be very useful for solving eigenvalue problems involving partially altered matrices. Reanalysis methods have received intense attention, and many papers have addressed the problem of changes in eigenvalues in locally modified systems [1–9]. Many of these papers used perturbation theory [1,4,7]. Fox and Kapoor [1] and Rogers [7] applied first-order differential to the eigenvalues and eigenvectors of the design variables. This kind of perturbation-based solution method is valid when there are relatively small adjustments to the design variables and when the initial values are assumed to be unchanged. However, if the changes to the design variables are large, these approaches generally offer low accuracy even when the perturbation order is increased.

Hirai et al. [2] demonstrated a way to obtain the exact eigenvalues and eigenvectors using only the degree of the modified part, and Parazzola et al. [6,10] applied this method to theoretical solutions for eigenvalues and eigenvectors in damped systems. These approaches use the eigenvalues and eigenvectors of the unmodified system to determine exact solutions on the basis of a fundamental formula having the same degree as the matrix representing the modified system. They remain valid regardless of the magnitude of the modifications. If eigenvalue problems can be reanalyzed using such a condensed equation, it will mean that matrices can be simplified to lower degrees, which is a very effective technique for reducing the calculation time. This fundamental equation is nonlinear and it can be used to solve a matrix equation consisting of rational functions. It is very time consuming to solve this kind of problem by trial and error and such a process does not indicate the order of the eigenvalues. Furthermore, there is little prospect of achieving

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a stable calculation process using the Newton—Raphson method or other methods in which the differential coefficients are set to appropriate values for local solutions and the initial estimate must be reasonably close to the actual solution (since the process becomes unpredictable if it is not). Even if an approximate range for the solution has been determined, it is difficult to find stable solutions using successive approximation methods such as reverse linear interpolation or polynomial approximation. Using one of these approaches may well provide some solutions, but it is well known that numerical calculations can provide solutions in a random order. When a solution has been found using, for example, the Newton—Raphson method, a deflation is set up and the procedure is repeated to determine the next eigenvalue. Thus, conventional methods have some fundamental weaknesses for solving these nonlinear equations.

Kashiwagi et al. have conducted a systematic study of compressed versions of the equations published by Hirai et al. [11–16]. They have addressed the problem of finding all the eigenvalues in a locally modified system, proposing combinations of the Durand–Kerner method [12] with Newton's method [16] and with rational functions [13] to find reliable solutions for eigenvalues in low-degree systems. They have also demonstrated that the Sturm sequence is useful for certain compressed, strongly nonlinear equations [14,15], discussed how to identify regions in which Sturm's eigenvalues exist, and described solutions using the Sturm sequence bisection method. The Sturm sequence method is a conventional, commonly used method for calculating eigenvalues; generally, it is most suitable for matrices that have been transformed into tridiagonal matrices [9]. The process of converting a matrix to tridiagonal form requires a considerable amount of calculation and generally represents more than half of the computation of identifying eigenvalues. The version of the Sturm sequence method developed by Kashiwagi et al. requires identifying all of the eigenvalues and eigenvectors of the unmodified system, but does not require transforming the matrix into tridiagonal form, so it represents a unique contribution to this field.

Many eigensolution problems seek just a few of the eigenvalues and their corresponding vectors from low-degree systems. Inverse power functions are an example of the iterative methods used to obtain solutions in such systems [17]. The inverse power method is a basic method for obtaining solutions in eigenvalue analysis; many approaches have been based on it. For example, subspace iteration [18,19] is a very commonly used method for determining eigenvalues. Thus, we have a nearly complete toolbox of methods for determining the eigenvalues of a locally modified system, but currently no method based on inverse power functions has been proposed.

This paper proposes just such a method based on inverse power method, using the DOF of only the modified part. This approach enables exact solutions for eigenvalues to be found quickly, regardless of the magnitude of the modification, using the degree of the condensed modified part. When the degree of that matrix is low, the calculation time is short, especially when the inverse power function is combined with a shift of the origin. All of the eigenvalues and eigenvectors of the unmodified system must be known, so for relatively small systems it is effective to determine just the first several eigensolutions (10 or less for practical systems), beginning from the smallest eigensolution. In the following sections, the theory and the algorithm of the proposed method are described and the effectiveness of this approach is demonstrated by numerically solving a typical eigenvalue problem.

2. Theory and algorithm for the inverse power method for locally modified systems

This section describes the theory of the inverse power method for locally modified systems and the theory of what is here termed *the shifted inverse power method for locally modified systems*. These theories enable the eigensolution of just the condensed version of the modified part to be exactly determined.

2.1. Inverse power method for locally modified systems

The general eigenvalue problem is as follows, assuming an $n \times n$ real symmetric matrix A and a positive real symmetric matrix B:

$$A\phi_i = \pi_i B\phi_i \tag{1}$$

where π_i is the ith eigenvalue, beginning with the smallest value and ϕ_i is the eigenvector corresponding to eigenvalue π_i. A and B take the following forms:

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & a_{22} & \cdots & a_{2n} \\ sym. & & \ddots & \vdots \\ & & & a_{nn} \end{bmatrix} \tag{2}$$

$$B = [b_{ij}] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ & b_{22} & \cdots & b_{2n} \\ sym. & & \ddots & \vdots \\ & & & b_{nn} \end{bmatrix} \tag{3}$$

If Φ is the mode matrix containing the eigenvectors for the problem expressed in Eq. (1)

$$\Phi = [\phi_1, \phi_2, \ldots, \phi_n] \tag{4}$$

then the following relationships hold:

$$\Phi^{\mathrm{T}} A \Phi = \Pi = \begin{bmatrix} \pi_1 & & & o \\ & \pi_2 & & \\ & & \ddots & \\ o & & & \pi_n \end{bmatrix} \tag{5}$$

$$\Phi^{\mathrm{T}} B \Phi = I \tag{6}$$

Here, I is the $n \times n$ identity matrix. From Eq. (5) we obtain

$$A^{-1} = \sum_i \frac{\phi_i \phi_i^{\mathrm{T}}}{\pi_i} \tag{7}$$

Free structural vibration and buckling are two eigenvalue problems that are typically analyzed by the FEM. For the free vibration problem, A is the stiffness matrix K and B is the mass matrix M. In buckling, A is the stiffness matrix K and B is the geometric stiffness matrix K_{G}.

Under a local modification, A is replaced by $A + \Delta A$ and B is replaced by $B + \Delta B$. Let us assume that ΔA and ΔB are as follows:

$$\Delta A = [\Delta a_{ij}] = \begin{bmatrix} 0 & & & \cdots & & & 0 \\ & \Delta a_{ii} & 0 & \cdots & 0 & \Delta a_{ij} & \vdots \\ & & 0 & \cdots & & 0 & \vdots \\ & & & \ddots & & \vdots & \\ & sym. & & & 0 & 0 & \vdots \\ & & & & & \Delta a_{jj} & \vdots \\ & & & & & & 0 \end{bmatrix} \tag{8}$$

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