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Topology optimization of pressure-actuated compliant mechanisms

Henry Panganiban, Gang-Won Jang *, Tae-Jin Chung

School of Mechanical and Automotive Engineering, Kunsan National University, Miryong 68, Kunsan, Jeonbuk 573-701, Republic of Korea

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A B S T R A C T

Topology optimization of a compliant mechanism under pressure input is presented by treating void regions with incompressible hydrostatic fluid. Since an input force is not imposed on one point, existing problem formulations such as attaching a spring on the node under the input force or constraining the input displacement are not valid for the present problem. Instead, to obtain the structural stiffness of a compliant mechanism, the mean compliance by the input pressure is considered. To deal with incompressibility, as an alternative to the mixed displacement–pressure formulation, displacement-based nonconforming finite elements are employed for both two- and three-dimensional problems. The effectiveness of the proposed approach is verified by designing grippers and stretchers.

1. Introduction

A compliant mechanism is a monolithic structure that acquires its desired motion from the elastic deformation of some or all its integral parts in contrast to a typical rigid-body type mechanism. It can be a stand-alone structure or a part of a system where the movement can be utilized for actuation of other components in the system. Recently, the widespread application of compliant mechanisms can be found in various fields such as micro electro-mechanical systems (MEMS) and robotics [1]. However, most of the existing techniques so far deal with single-point input force to actuate the mechanism. As the areas of application of this technology continuously expand, especially in MEMS, it is desirable that this technique be extended to other possible problems such as those involving pressure as the actuating force.

The design of a compliant mechanism concerns major consideration of the kinematical functionality or flexibility and stiffness of the structure for the efficient transfer of input to output work. In Sigmund [2], displacement constraint at the input point was added to impose structural stiffness and control the maximum stress level. In Frecker et al. [3], flexibility and stiffness of a compliant mechanism were both taken into account in the multi-criteria topology optimization. This was made possible by employing the objective function as the ratio between mutual energy and strain energy of the system. Topology optimization of a compliant mechanism has also been extended to thermal and electromechanical applications [4–6].

Topology optimization with a pressure load is a typical design-dependent load problem wherein the direction and location of the load vary with the change in shape of the pressurized boundary. One way to find optimum topology for a continuum structure under a design-dependent load is by considering the shape of the loaded boundary and defining the load acting on it [7–9]. Another approach is to use fictitious thermal loads to simulate the design-dependent loads instead of defining parameterized loaded boundaries [10]. In Sigmund and Clausen [11], a technique using incompressible material had been developed where, to deal with incompressibility, the displacement–pressure mixed formulation was employed. In this approach, pressure is imposed on an external boundary of a design domain and is transferred to its corresponding boundary in the structure using the incompressibility of the material. In this case, the load is not numerically design-dependent. Thus, instead of parameterizing pressure-loaded surfaces, the formulation allows for the transfer of pressure from an external boundary to the structure by defining void phase as hydrostatic incompressible fluid.

Obviously, the application of the mixed formulation in [11] is to overcome the difficulty caused by the incompressible behavior of a material. An alternative approach without introducing a pressure as an additional field variable can be formulated with the use of nonconforming finite elements [12]. The use of nonconforming elements for topology optimization can be traced back to [13,14]. These studies highlighted the ability of nonconforming finite elements to overcome the common checkerboard problem and locking phenomenon in contrast to the use of low-order conforming counterpart. The Poisson locking-free property of nonconforming elements [15] can deal with problems involving incompressible material based on the pure-displacement formulation. Using this property, Jang and Kim [16] employed

* Corresponding author. Tel.: +82 63 469 4725; fax: +82 63 469 4727.
E-mail address: gangwon@kunsan.ac.kr (G.-W. Jang).

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nonconforming elements for solving mean compliance minimization problems with incompressible material.

The objective of this investigation is to formulate the topology optimization problem of compliant mechanism with a pressure input load in the framework of displacement-based nonconforming elements. In the case of a pressure input load, the stiffness of a compliant mechanism cannot be imposed by attaching an input spring or constraining the displacement of one prescribed point. Instead, the input displacement should be constrained in the form of integration over the pressurized boundary of the compliant mechanism. Adopting the idea of using hydrostatic fluid for transmitting pressure to the boundary of a structure [11], the integration of input displacement can be performed on the boundary of a nondesign hydrostatic fluid domain regardless of the varying pressurized boundary of the compliant mechanism. Because the pressure is constant over the pressurized hydrostatic fluid region, constraining the integrated value of the input displacement can be regarded as constraining the mean compliance of the system. Thus the upper bound for the input displacement constraint can be decided by considering the mean compliance of the compliant mechanism.

After showing the validity of nonconforming elements for pressure load problems through several benchmark problems, pressure-actuated grippers and stretchers are designed by following the proposed optimization formulation.

2. Analysis of pressure load problems using incompressible medium

2.1. Material interpolation scheme for solid, incompressible fluid and void

For the pressure load problem, the solid isotropic material with penalization (SIMP) approach is modified to facilitate different material representation; the parameterization between materials and design variables is expressed in terms of the bulk modulus, K, and shear modulus, G. For element e,

$$K(\rho_e) = K_e = K_{\text{fluid}} + \rho_e^p (K_{\text{solid}} - K_{\text{fluid}}), \tag{1a}$$

$$G(\rho_e) = G_e = G_{\text{fluid}} + \rho_e^p (G_{\text{solid}} - G_{\text{fluid}}), \tag{1b}$$

with

$$0 \le \rho_e \le 1,$$

where the quantity with subscript fluid and solid refer to material properties for incompressible fluid and solid, respectively. In Eq. (1), p (≥ 3) is the penalty exponent to push the optimization solution towards a 0–1 design. For plane strain problems, the bulk modulus of material is evaluated as $K = E/2(1+\nu)(1-2\nu)$ and, for three-dimensional problems, $K = E/3(1-2\nu)$. The shear modulus is $G = E/2(1+\nu)$ regardless of the dimension. Incompressibility is imposed by setting a large value for K_{fluid} (10–100 times larger than K_{solid}).

To deal with problems involving three material states (solid, incompressible fluid, void), one needs to introduce two design variables $\rho_{1,e}$ and $\rho_{2,e}$ at every element such that

$$\begin{aligned} K(\rho_{1,e}, \rho_{2,e}) = {} & (K_{\text{solid}} - K_{\text{void}})(\rho_{1,e})^p [1 - (\rho_{2,e})^q] \\ & + (K_{\text{fluid}} - K_{\text{void}})(\rho_{2,e})^p [1 - (\rho_{1,e})^q] + K_{\text{void}}, \end{aligned} \tag{2a}$$

$$G(\rho_{1,e}, \rho_{2,e}) = (G_{\text{solid}} - G_{\text{void}})(\rho_{1,e})^p [1 - (\rho_{2,e})^q] + G_{\text{void}}, \tag{2b}$$

with

$$0 \le \rho_{1,e}, \rho_{2,e} \le 1,$$

where $G_{\text{fluid}} = G_{\text{void}}$ is used. In Eq. (2), $\rho_{1,e}$ determines whether the element is fluid or void if $\rho_{2,e} = 1$, and whether it is solid or void if $\rho_{2,e} = 0$. To impose incompressibility, as in the case of two-material parameterization in Eq. (1), the value of K_{fluid} is set large while the value of $G_{\text{fluid}} = G_{\text{void}}$ is chosen to be very small.

2.2. Displacement-based nonconforming elements

In this work, to deal with incompressible material, the analysis is formulated with the displacement-based nonconforming elements. The basic properties of nonconforming elements are briefly given in this subsection (see [12,15] for detailed mathematical descriptions). The convergence of nonconforming elements is given as

$$\|u - u_h\|_{1,h} \le Ch\|u\|_2, \tag{3}$$

where u and u_h denote an exact solution and a finite element solution by nonconforming elements, respectively, and h is a characteristic element size. Since nonconforming elements have discontinuity along element edges, element-wise calculation of the energy norm in Eq. (3) is carried out. Note that C is a constant with respect to the material property, by which the present nonconforming elements can be free from Poisson locking for incompressible material with $\nu \approx 0.5$.

For the convergence in Eq. (3), the displacement continuity of nonconforming elements is imposed only at the midpoints of the element edges for two-dimensional problems and at the centroids of the element faces for three-dimensional problems. Thus the nodes are not located at the vertices of an element as in conforming finite elements (see Fig. 1(a) for the location of nodes of nonconforming elements).

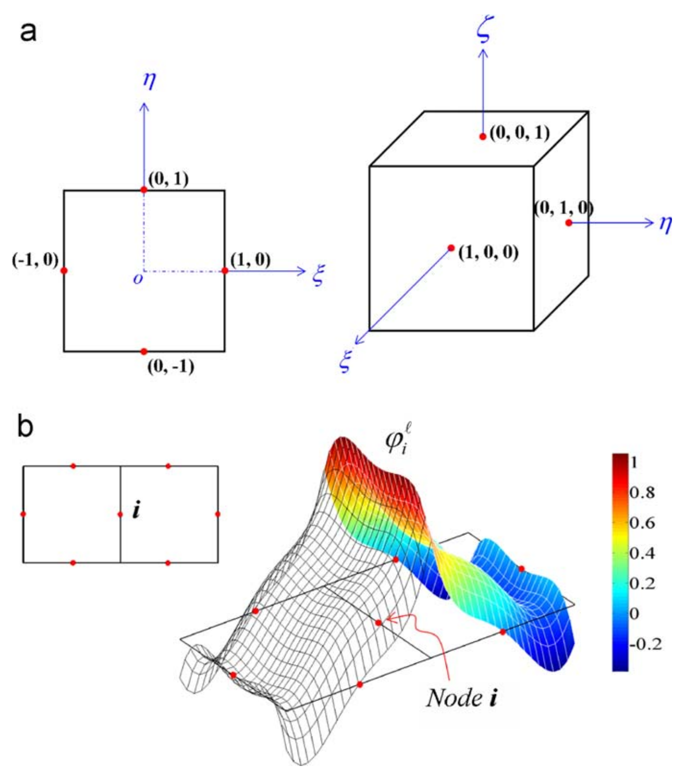

Fig. 1. (a) Two- and three-dimensional nonconforming elements and (b) two-dimensional shape functions associated with node i.