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Solving inverse couple-stress problems via an element-free Galerkin (EFG) method and Gauss–Newton algorithm

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ABSTRACT

This paper focuses on the identification of constitutive parameters in the couple stress problem. The direct problem is modeled by element-free Galerkin method (EFGM), thus the inconvenience that may be caused by C^1 continuity requirement in the implementation of FEM can be avoided, and the sensitivity analysis that is required for the solution process of the inverse problem can be carried out conveniently. The inverse problem is solved via the Gauss–Newton technique. The proposed method is verified in the cases of slight and strong regional inhomogeneity. The effects of initial guesses, noisy data and location of the measured points on the solutions are investigated, and satisfactory results are achieved.

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1. Introduction

The existence of couple stress was originally postulated by Voigt in 1887. In 1909, the brothers E. and F. Cosserat first set up a framework of couple stress theory which has been further developed since then [1–4]. Couple stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume [5]. Accordingly, a group of variables including moments, curvatures, and characteristic length are introduced within a continuum framework [3].

One important application of couple stress theory was to describe the materials with microstructures, such as the materials with granular [6], fibrous [7] and lattice structures [8]. In the addition, in the some cases where the size effects have to be taken into account [9], the theory was employed to explain the variation of hardening behavior [10], and local singularities [11].

The study of this paper is motivated by a question that if a continuum couple stress model is adopted, how to determine relevant constitutive parameters, including the so-called characteristic length ℓ?

One of the solutions is to treat this issue as an inverse problem with unknown constitutive parameters. This inverse problem can be investigated under the framework of inverse problems in elasticity for which a comprehensive review was given by Bonnet [12]. If the sufficient 'measurement' message, such as the displacements, strains etc. is provided, all the unknowns are able to be determined analytically or numerically. In comparison with the previous work based on the classical elasticity, the parameters identification of the inverse couple stress problem includes both constitutive parameters appearing in the classical model and those additional items describing the constitutive relationship of couple stress. To the best of the authors' knowledge, it seems there are no reports directly relevant to this matter.

Since the displacement that is usually reliable and is easy to measure [13], it is employed as 'measurement' message in this paper. We propose a numerical model that consists of two parts, one is concerned with the direct problem formulated by element-free Galerkin method [14], and the implementation of sensitivity analysis; the another is for the description of inverses problem that is treated as an optimization problem solved by the Gauss–Newton technique, the major issues concerned in this part include the combined identification, regional inhomogeneity, and computing accuracy with the consideration of noisy 'measurement' message and location of measured points.

2. Governing equations for direct couple stress problems

For plane couple stress problems in the absence of body forces and couples, the equilibrium equations are given by [15]

$$\begin{aligned} &\frac{1}{2}(\sigma_{ij}+\sigma_{ji})_{,j}+\frac{1}{2}(\sigma_{ij}-\sigma_{ji})_{,j}=0 \\ &\mu_{i,i}+\alpha_{ij3}\sigma_{ij}=0 \end{aligned} \qquad \text{in} \quad \Omega \tag{1}$$

where σ_{ij} stands for the Cauchy component of the stress tensor, μ_i denotes the component of moments, α_{ij3} is the permutation symbol, subscript i and j range form 1 to 2.

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The relationship of displacement and strain is described by [15]

$$\{\varepsilon\}=[L]\{u\} \tag{2}$$

where $\{u\}=\{u_1,u_2\}^T$ represents the vector of displacement, $\{\varepsilon\}=\{\varepsilon_{11},\varepsilon_{22},\varepsilon_{12},\kappa_1,\kappa_2\}^T$ represents strain vector, κ_i designates curvature corresponding to μ_i and is specified by

$$\kappa_i=\theta_{,i},\quad i=1,2. \tag{3}$$

where θ is a microrotation defined by

$$\theta=\frac{1}{2}\alpha_{ij3}u_{j,i} \tag{4}$$

$$[L]^T=\begin{bmatrix} \dfrac{\partial}{\partial x_1} & 0 & \dfrac{\partial}{\partial x_2} & -\dfrac{\partial^2}{2\partial x_1\partial x_2} & -\dfrac{\partial^2}{2\partial x_2^2} \\ 0 & \dfrac{\partial}{\partial x_2} & \dfrac{\partial}{\partial x_1} & \dfrac{\partial^2}{2\partial x_1^2} & \dfrac{\partial^2}{2\partial x_1\partial x_2} \end{bmatrix} \tag{5}$$

The boundary conditions are specified by [15]

$$\begin{matrix} u_i=\tilde{u}_i \\ \theta=\tilde{\theta} \end{matrix}\quad x\in\Gamma_u \tag{6}$$

$$\begin{cases} \sigma_{ij}n_j=T_i^0 \\ \mu_j n_j=q_i^0 \end{cases}\quad x\in\Gamma_\sigma \tag{7}$$

where $\{\tilde{u}\}$ and $\tilde{\theta}$ are the prescribed values of $\{u\}$ and θ on Γ_u, T_i^0 and q_i^0 are the prescribed vectors of traction and moment on Γ_σ, n_j denotes the unit outside normal on the boundary, $\Gamma_u+\Gamma_\sigma=\Gamma$ designates the whole boundary of Ω, x represents a vector of coordinates. Subscripts u and σ refer to displacement and stress, respectively.

The constitutive relationship is described by

$$[D]=\begin{bmatrix} \dfrac{E}{1-v^2} & \dfrac{Ev}{1-v^2} & 0 & 0 & 0 \\ \dfrac{Ev}{1-v^2} & \dfrac{E}{1-v^2} & 0 & 0 & 0 \\ 0 & 0 & \dfrac{E}{2(1+v)} & 0 & 0 \\ 0 & 0 & 0 & 4\beta & 0 \\ 0 & 0 & 0 & 0 & 4\beta \end{bmatrix}\ \text{for the plane stress problem [15]} \tag{8}$$

where $\beta=\ell^2G=\ell^2(E/2(1+v))$ is called the curvature modulus, E, v and ℓ are Young's modulus, Poisson's ratio and character length, respectively.

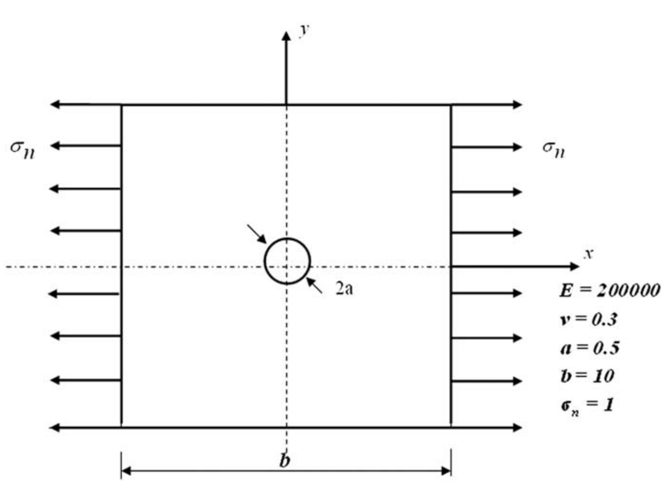

Fig. 1. A circular hole in a uniform tension field.

[D] can be decomposed into

$$[D]=b_1[H_1]+b_1[H_2]+b_3[H_3] \tag{9}$$

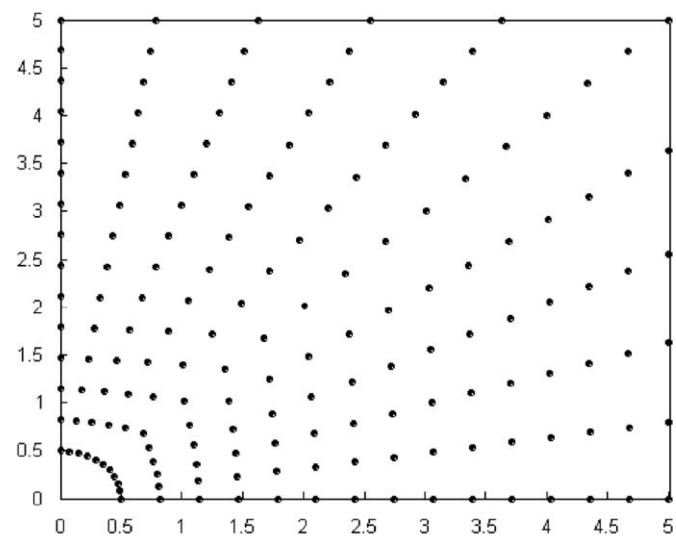

Fig. 2. Nodal arrangement (99 nodes).

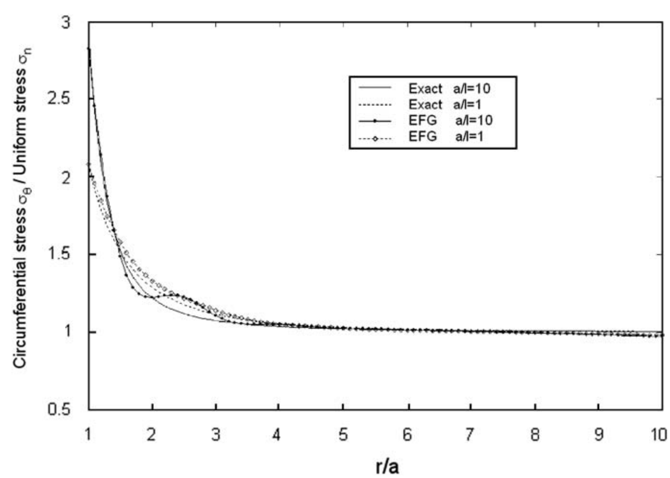

Fig. 3. The comparison of σ_θ/σ_n at θ=90°.

Table 1
The comparison of σ_θ/σ_n at θ=90°.

σ_θ/σ_n at θ=90°						
r/a	$\ell=0.5$			$\ell=0.05$		
	Exact	EFG	% error	Exact	EFG	% error
1.00	2.0666	2.0778	0.54	2.9130	2.8209	3.16
1.10	1.9129	1.9553	2.22	2.4220	2.4552	1.37
1.20	1.7853	1.8437	3.27	2.0737	2.1372	3.06
1.30	1.6796	1.7434	3.80	1.8279	1.8696	2.28
1.40	1.5916	1.6545	3.95	1.6522	1.6535	0.078
1.50	1.5181	1.5769	3.87	1.5241	1.4876	2.39
1.60	1.4562	1.5101	3.70	1.4287	1.3683	4.23
1.70	1.4038	1.4533	3.53	1.3563	1.2899	4.89
1.80	1.3591	1.4055	3.42	1.3001	1.2452	4.22
1.90	1.3207	1.3654	3.39	1.2559	1.2256	2.41
2.00	1.2876	1.3318	3.43	1.2206	1.2226	0.16

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