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Multi-scale modelling of sandwich structures using the Arlequin method Part I: Linear modelling

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ABSTRACT

The paper presents an Arlequin based multi-scale method for studying problems related to the mechanical behaviour of sandwich composite structures. Towards this end, different models are mixed and glued to each other. Several coupling operators are tested in order to assess the usefulness of the proposed approach. A new coupling operator is proposed and tested on the different glued Arlequin zones. A free–clamped sandwich beam with soft core undergoing a concentrated effort on the free edge is used as a typical example (benchmark) in the validation procedure. Numerical simulations were conducted as the preliminary evaluation of the various coupling operators and the discrepancies between local and global models in the gluing zone have been addressed with sufficient care.

1. Introduction

The use of sandwich structures continues to increase rapidly for applications ranging from satellites, aircraft, ships, automobiles, rail cars, wind energy systems and bridge construction to mention only a few. One challenge in these applications is to understand in detail how different damages influence the structure and what their behaviour in-service is. In this situation, an accurate solution is often required and calculations must be performed on a finely discretized model of the structure (in the micro-level).

Despite advances in computational techniques and computing power, direct simulation of these materials is still not a viable option. Thus, there is a need for accurate and computationally efficient techniques that take into account the most important scales involved in the goal of the simulation while permitting the analyst to choose the level of accuracy and detail of description desired.

Generally, finite elements (FEs) modelling of structures is very tedious when the finer details need to be captured or when non-linear calculations are carried out. To overcome these difficulties and make the FE methods more flexible, important innovative and efficient numerical methods have been developed. Let us mention in particular the sequential adaptation method, the multigrid (MG) method, the partition of unity finite element method (PUFEM) and the extended finite element method (XFEM). The sequential adaptation method consists of carrying out structure modelling sequence by sequence. At the end of each sequence, an error is estimated and the discretization (size of mesh and/or degrees of freedom of the shape function) is refined as long as the estimated error is higher than the prescribed value. In this category, one could mention the h-adaptation, the p-adaptation and the hp-adaptation which is a combination of these two methods.

The MG method is a family of techniques for solving differential equations using a hierarchy of discretizations. The idea behind is similar to extrapolation between coarser and finer grids and can treat arbitrary regions and boundary conditions. MG can be applied in combination with any of the common discretization techniques.

Since the introduction of singular enrichment method [1] using a cut-off function for a mesh dependent on the domain geometry, different approaches had been analysed such as PUFEM [2] and GFEM (generalized finite element method) [3]. Inspired by PUFEM, the XFEM was introduced by Moës et al. [4]. The idea of XFEM consists of enriching the basis of the classical FE method to take into consideration the discontinuity of the displacement field.

All these approaches are essentially monomodel and may either lack flexibility or relevance to address the above issues. Recent hierarchical global–local strategies of global–local techniques that allow the superposition of different mechanical models are the *s*-version method by Fish [5–7] and the Arlequin method by Ben Dhia et al. [8–12]. The *s*-version method is a multilevel solution scheme where each level is discretized using an FE mesh of arbitrary element size and polynomial order. It superimposes additional local and refined

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meshes to an existing global one, thus allowing different modelling in the superimposed meshes. Like the *s*-method, the Arlequin method [10,12] aims at creating a multi-model framework. The models are crossed and glued to each others. In addition, the fact that models are locally crossed with each other theoretically allows the coexistence of substantially different mechanical and numerical models. Iteration of the crossing process [9] (by taking care of multiple gluing zones) can potentially lead to some relevant multiscale models. We refer to [7,13] for a comprehensive review of global–local techniques for composite laminates and to [14,15] for various aspects of reliability, convergence and accuracy of global–local techniques.

In this manuscript, an attempt is made to apply the Arlequin method for the modelling of sandwich structures and particularly to capture local effects in these structures. Different FE models have been glued to others in order to see the relevance and the capabilities of the approach. These are illustrated through typical dedicated applications like

1. Locally refined models (zoom): the 2D coarse FE model coupled to a 2D refined FE model.
2. Link structure models (sub-structuring): the 2D FE models coupled to a 1D (zig-zag or beam) model and 2D FE model coupled to an analytical model.

2. Arlequin method: formulation and implementation issues

Following the Arlequin framework [9], the domain Ω representing the mechanical system is partitioned into two overlapping sub-zones Ω_1 and Ω_2 (Fig. 1). The resulting intersecting zone constitutes the gluing zone S. The internal and external virtual works are expressed as

$$\delta P_i^{\text{int}}(u_i) = -\int_{\Omega_i} \alpha_i \delta\varepsilon(u_i)\sigma(u_i), \tag{1}$$

$$\delta P_i^{\text{ext}}(u_i) = \int_{\Omega_i} \beta_i \delta u_i f_i, \tag{2}$$

where u_i, δu_i and f_i are, respectively, the displacement, the virtual displacement and the external force in Ω_i. To avoid considering twice the energy of the total system in the covering zone, the virtual work, associated to each zone, is balanced by some weighting or blending functions. The latter form a partition of unity on the whole domain. These functions are represented by α_i for the internal work and by β_i for the external work. α_i and β_i are assumed to be positive piecewise continuous functions in Ω_i and satisfy the following equalities:

$$\begin{cases} \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1 & \text{in } \Omega_1 \backslash S,\ \Omega_2 \backslash S, \\ \alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1 & \text{in } \Omega_1 \cap \Omega_2. \end{cases} \tag{3}$$

It is clear that the Arlequin solution depends on the choice of α_i and β_i. The operational choice [9] of these functions consists of relating their values to the relative local refinement of the associated models. The natural way to treat the gluing volume of displacement fields consists of introducing the Lagrange multiplier field belonging to the dual of the space of the admissible displacement fields restricted to S (see [9]).

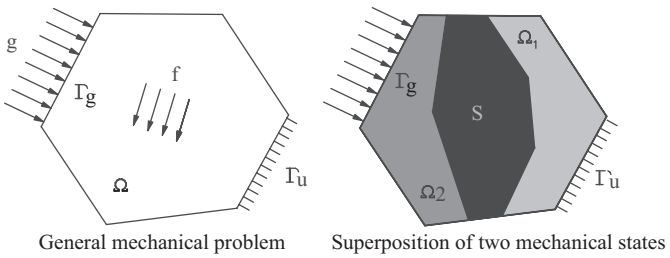

Fig. 1. The Arlequin method in a general mechanical problem.

The virtual work equation takes the following form:

$$\delta P_i^{\text{int}}(u_i) + \delta P_i^{\text{coup}}(u_i) + \delta P_i^{\text{ext}}(u_i) = \delta P_i^{\text{acc}}(u_i), \quad \forall \delta u_i \in K.A. \tag{4}$$

where *K.A.* in Eq. (5) holds for kinematically admissible, δP_i^{acc} and δP_i^{coup} are, respectively, the inertial terms and the virtual coupling work. δP_i^{coup} is described by the following form:

$$\delta P_i^{\text{coup}}(u_i) = (-1)^i C_i(\lambda, \delta u_i). \tag{5}$$

C_i denotes the coupling operator which is a function of λ and δu_i. The expression $(-1)^i$ is introduced to give C_1 and C_2 opposite signs.

Note that if $(u_1, u_2$ and $\lambda)$ denote the solution of a given structural problem, the coupling virtual work should be the same for *any given set of Langrange multiplier* λ:

$$C_1(\delta\lambda, u_1) - C_2(\delta\lambda, u_2) = 0. \tag{6}$$

The FE approximations are used to solve Eqs. (4) and (6). Thus, the displacements u_i and the Lagrange multipliers λ are discretized as follows:

$$\begin{cases} u_i = \langle N_i \rangle \{u_i^{\text{e}}\}, \\ \lambda = \langle N_{\text{c}} \rangle \{\lambda^{\text{e}}\}, \end{cases} \tag{7}$$

where u_i^{e} and λ^{e} denote, respectively, the elementary displacement vector and the elementary Lagrange multiplier field. N_i and N_{c} are, respectively, the shape functions associated to displacement and to the Lagrange multiplier fields. Finally, the discrete formulation of the analysed problem is derived and expressed as

$$\begin{bmatrix} K_1 & 0 & {}^{\text{t}}C_1 \\ 0 & K_2 & -{}^{\text{t}}C_2 \\ C_1 & -C_2 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \Lambda \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 0 \end{bmatrix}, \tag{8}$$

where

$$(K_i)_{jk} = \int_{\Omega_i} \alpha_i \sigma(N_i^j) : \varepsilon(N_i^k), \tag{9}$$

$$(F_i)_j = \int_{\Omega_i} \beta_i f(N_i^j). \tag{10}$$

It is clear that the construction of the coupling matrix is central in the application of the Arlequin method and essentially, when different models are mixed together like when a 3D model is coupled with a 2D one or when a 2D model is coupled with a 1D model. In the present study, the Lagrange multiplier field is always established on the coarse mesh (see [9]). Bearing this in mind, three coupling operators are considered for the present study and thereafter evaluated:

- H^1 coupling

$$C_i = \int_S \lambda \cdot u_i + \ell^2 \varepsilon(\lambda) : \varepsilon(u_i). \tag{11}$$

- L^2 coupling

$$C_i = \int_S \lambda \cdot u_i. \tag{12}$$

- L^2_{p} coupling

$$C_i = E_{\text{p}} \int_S \lambda \cdot u_i. \tag{13}$$

ℓ is a strictly positive parameter homogeneous to a length and E_{p} is a kind of a weight parameter which depends on the Young modulus in the coupling zone.