Available online at www.sciencedirect.com
ScienceDirect
Finite Elements in Analysis and Design 43 (2007) 1082–1098
FINITE ELEMENTS IN ANALYSIS AND DESIGN
www.elsevier.com/locate/finel

On honeycomb representation and SIGMOID material assignment in optimal topology synthesis of compliant mechanisms

Rajat Saxena, Anupam Saxena*,[1]

Mechanical Engineering, Indian Institute of Technology, Kanpur 208016, India

Received 24 October 2006; received in revised form 26 July 2007; accepted 20 August 2007
Available online 1 October 2007

Abstract

This paper introduces a new honeycomb based domain representation and SIGMOID material function to model continuum topology optimization domains with fixed grids. The two-point connectivity ensured with hexagonal cells is shown to circumvent singularities related to checkerboard patterns and point flexures that are typically observed with one-point connected rectangular cells. The novel SIGMOID function is developed to assign the 'solid' material for very low values of design variables (probabilities) and 'void' material for those further lower than the threshold, thus encouraging the binary material assignment. The performance of the SIGMOID function is compared with the previously proposed SIMP and PEAK material assignment functions. It is shown that both SIMP and PEAK functions can be over-restrictive in that the material is assigned only for probabilities close to one. The proposed modeling is suitable for topology optimization objectives wherein the number of constraints is large and gradient-based algorithms are chosen. Numerous examples on material layout determination for compliant mechanisms are solved with flexibility-stiffness and flexibility-strength multi-criteria formulations to illustrate the essence of this paper.

Keywords: Topology optimization; Honeycomb parameterization; Compliant mechanisms; Flexibility-stiffness; Flexibility-strength; SIMP; PEAK; SIGMOID

1. Introduction

A topology design problem entails determining the optimal *material distribution* or *layout* in a design region which can be represented using a fixed array of sub-regions (unit cells), which in turn are approximated using *finite elements*. Each cell (or finite element) is assigned a design variable that models its material state. Design variables are related to the physical attributes of a cell or element, like the cross-section or elastic modulus. A *zero* value of a design variable represents *no-material* (void) state while a *unit* value symbolizes the *full-material* state. The material states are determined using an optimization method in conjunction with the finite element analysis that evaluates the functional objective (e.g., maximizing stiffness or compliance) of the design. Ideally, an optimal topology should solely consist of solid and void states in the so called "black and white" design, implying that "integer" values of only 0 or 1 are sought for design variables. However, if one is restricted to using *gradient-based* optimization, for instance in problems where the number of constraints are very large and need to be satisfied in each sub-problem, to facilitate the computation of objective and constraints' gradients, the design variables are *relaxed* as continuous functions bounded between "0" (void state) and "1" (solid state).

Parameterization of a design region with a finite grid involves its representation in two levels: (a) *physical* or *geometric level* that entails discretizing a domain into sub-regions (cells) of known shapes and (b) *mathematical* that requires modeling the material state of each cell as a function of its design variable. Any parameterization scheme should ensure that (i) at any step in optimization, the global finite element stiffness matrix should always be non-singular for objective and constraint values and/or their gradients to be unique, (ii) the variables or sub-regions span the entire design region to ensure the participation of all sites and (iii) the optimal topology should be manufacturable. In view of (i), singularity of the stiffness matrix may be partly avoided by using a small but positive parameter $\in$ as the lower bound for design variables.

* Corresponding author.
E-mail address: anupams@iitk.ac.in (A. Saxena).
[1] Currently, visiting faculty in Cornell University.

0168-874X/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.finel.2007.08.004

Also, the sub-regions (or finite elements) should be *well connected*. The requirement (ii) may not be essential in that the optimal topologies may be determined using a relatively coarse mesh wherein some sites in the domain may not participate in topology optimization (e.g., when using truss/frame elements). However, mesh dependency may exist and also, the search may be limited to a few of many possible topologies. Finally, from the manufacturing viewpoint (iii), appropriate functions should be used to model the relationship between the design variables and respective material states such that the optimal topology is realizable, ideally, without requiring user interpretation.

Physical representation of a two-dimensional design region via a fixed finite element mesh may involve the use of either *discrete* (truss/frame) elements or *continuum* (triangular/quadrilateral) elements. With the former, *full* or *partial* ground structures are often used. A full ground structure [1] is an assemblage wherein every node is connected via a truss or frame element to every other node in the domain. In view of (ii) above, attempt is made to map all sites in the design region into the finite element mesh. A drawback, however, considering (iii) above, is that the single-piece optimal topologies may be difficult to manufacture as some overlapping elements may exist. A partial truss ground structure, on the other hand, ensures no element overlap but at the expense of losing numerous sites in the finite element mesh and thus numerous possible topologies. Discrete elements that assume non-existing states (their cross-sections near the lower bound $\in$) at convergence still contribute, though insignificantly (depending on how small $\in$ is), to the structural stiffness matrix. Complete extraction of such elements from the optimal topology may cause the resultant global stiffness to become singular. This may be the case when using truss elements as their ends are modeled as pin joints. The optimal topology resulting from the physical removal of *mathematically non-existing* elements may retain some dangling trusses with only one end connected. Frame elements in a partial ground structure, on the other hand, are rigidly connected with non-zero torsional stiffness at both their ends, and thus do not pose the aforementioned difficulty. With frame elements in a partial ground structure, the extracted optimal topologies can be manufactured so long as the cross-sections (as design variables) are larger than the permissible manufacturing limit. Thus, limited only by the inability to explore a comprehensive set of possible topologies, material layout design of compliant mechanisms with partial frame ground structures has been accomplished by Saxena and Ananthasuresh [2], Canfield and Frecker [3] and others with linear deformation analysis and Saxena and Ananthasuresh [4] using geometrically large deformation to achieve prescribed nonlinear force-displacement characteristics and curved output paths. Some recent works on topology optimization of compliant mechanisms using geometrically nonlinear frame elements are by Saxena [5,6], Rai et al. [7] and Zhou and Ting [8,9].

Continuum sub-region models have additional advantages over the discrete ones in that the former span the design region more comprehensively satisfying criterion (ii) of the design region parameterization above and shape and topology optimization get performed simultaneously. Initially conceived by Bendsøe and Kikuchi [10] who used *homogenization* to numerically approximate the stiffness–properties of a unit rectangular cell with a hole, continuum models have evolved from the use of layered microstructures [11] to that of SIMP (solid isotropic material with penalization) models [12–15]. In homogenization, a hole in a unit cell can either be rectangular or elliptical in shape. The design variables that model the length and width of a rectangular hole, or the axes of an elliptical one (say μ_1 and μ_2 in both cases, called the void parameters) represent the material state of the cell. With μ_1 and μ_2 both 1, the hole occupies the cell area, and the cell is in its no-material state. When $\mu_1 = \mu_2 = 0$, the cell assumes the solid state. Stiffness of the cell is computed numerically in terms of the void parameters. The second type of continuum models, the layered microstructures are obtained in a repetitive process in each step of which, a new layering of given direction is added to the microstructure. The number of times the steps are performed depends on the rank of the microstructure. Rank 1 microstructure is obtained by arranging alternately thin layers of stiff and relatively flexible materials in the proportion, μ_1 and $1 - \mu_1$ in the direction n_1. Rank 2 microstructure is obtained likewise using layers of stiff and Rank 1 materials in the ratio μ_2 and $1 - \mu_2$ in the direction n_2. Material stiffness of such microstructures is determined as functions of μ_i using *smear-out* or *quasi-convexification* techniques. A detailed review of topology optimization with fixed grids and continuum elements is provided by Eschenauer and Olhoff [16].

More commonly used continuum models in recent times are the rectangular SIMP cells, advantages over the *hole-in-cell* and *layered microstructures* being (i) the reduced number of design variables per cell and (ii) avoidance of numerical procedures or quasi-convexification methods to estimate the stiffness properties. Note that for $\mu_1 = \mu_2 = \mu$, the number of design variables in homogenization and SIMP parameterization is the same. Design sub-regions as rectangular cells are directly approximated using four-node finite elements wherein the elasticity tensor $\mathbf{E}_i$ for each cell i is given as

$$\mathbf{E}_i = x_i^{\eta}\mathbf{E}_0, \tag{1}$$

where $\mathbf{E}_0$ is the tensor for the solid element, x_i is the cell design variable ($\in \leqslant x_i \leqslant 1$) representing the material state and η is the user-specified penalization exponent. The SIMP parameterization has been applied to a range of topology optimization problems. For minimum mean compliance of stiff structures, Bendsøe [12], Zhou and Rozvany [13], Mlejnik and Schirrmacher [14], and others have used linear deformation analysis while Buhl et al. [17] have employed nonlinear deformation model. For compliant mechanisms' design, Sigmund [18] has used linear deformation to maximize the geometric advantage. Pedersen et al. [19] have employed geometrically nonlinear analysis to achieve prescribed force displacement relationship. Sigmund [38,39] has extended the use of SIMP parameterization to topology optimization of compliant mechanisms for applications with multiple output ports and materials using multi-physics analysis. As opposed to discrete elements wherein the variables representing the respective cross-sections