Finite Elements in Analysis and Design 96 (2015) 41–50

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Nonlinear elastic buckling and postbuckling analysis of cylindrical panels

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ARTICLE INFO

Article history:
Received 18 June 2014
Received in revised form
10 December 2014
Accepted 10 December 2014
Available online 26 December 2014

Keywords:
Nonlinear buckling and postbuckling analysis
The arclength method
The branch-switching method
Critical points
Secondary paths

ABSTRACT

This paper revisits the buckling analysis of a benchmark cylindrical panel undergoing snap-through when subjected to transverse loads. We show that previous studies either overestimated the buckling load and identified a false buckling mode, or failed to identify all secondary solution branches. Here, a numerical procedure composed of the arclength and branch switching methods is used to identify the full postbuckling response of the panel. Additional bifurcation points and corresponding secondary paths are discovered. Parametric studies of the effect of the rise, thickness, and boundary conditions of the panel on the buckling and postbuckling responses are also performed.

1. Introduction

Cylindrical shells are widely used in aerospace, mechanical, and civil engineering applications as structural components in aircraft, tanks, pipelines, and offshore platforms. These structures have efficient load-carrying capabilities but exhibit high risk of buckling failures.

Early studies on the buckling analysis of cylindrical shells used the classical buckling theory to approximate the bucking loads and mode shapes by assuming membrane prebuckling stress states [1–4]. This approach ignores bending effects before buckling and usually overestimates buckling loads. Later, more rigorous buckling analyses were performed with the consideration of linear prebuckling deformations [5–8] and nonlinear prebuckling deformations [9,10], but did not focus on postbuckling responses. Koiter [11] proposed a perturbation approach to conduct initial postbuckling analysis, which was later adopted by many researchers [12–15]. These methods are typically valid only in the vicinity of critical points. Potier-Ferry and coworkers [16–19] extended Koiter's idea and developed an asymptotic-numerical method to compute nonlinear postbuckling responses.

Other numerical approaches widely used to perform nonlinear postbuckling analysis of shell structures are path following schemes. Among them, the Newton–Raphson methods were initially attractive for solving large nonlinear systems but they usually lose convergence at limit points and cannot trace the unstable equilibrium paths. Some of these disadvantages were solved by replacing the load control with displacement control [20,21], but this approach still fails to track the whole postbuckling path beyond a displacement limit point. Riks [22] proposed a more efficient arclength method that can trace the entire (stable and unstable) postbuckling equilibrium paths. Modified versions were later proposed by Crisfield [23] and Tsai et al. [24] to handle more complicated postbuckling behavior.

Despite the great progress made in the path following approaches, some features of the postbuckling behavior still remained unnoticed. A circular cylindrical panel, studied by Sabir [25], was afterwards used by many researchers [26–35] as a benchmark example to demonstrate the capability of shell or shell-like elements in simulating large deformations buckling and postbuckling processes. All these researchers successfully identified the limit-point buckling and the corresponding symmetric postbuckling responses by utilizing path following methods. However, these studies did not correctly identify the physical buckling behavior of this panel. Recently, Wardle et al. [36,37] found using the asymmetric meshing technique (AMT) that a bifurcation buckling in asymmetric mode exists before the first limit point on the equilibrium path.

In this work, an arclength method combined with a branch-switching method [38,39] is used to perform the nonlinear buckling and postbuckling analysis of cylindrical panels. For the benchmark example, we find that *two previously undetected pairs of bifurcation points and consequently two other pairs of secondary paths exist.* A small

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http://dx.doi.org/10.1016/j.finel.2014.12.001
0168-874X/

interval of one secondary path is stable, while the other equilibria on this path and all states on the other path are unstable. Intervals of stable equilibria identified on secondary paths, while not reachable through a continuous stable path, are still important: perturbations in the system may lead to dynamic jumps to these states. The identification of additional unstable equilibria also reveals that the degree of instability of the system is higher than what researchers previously found.

The numerical approach used in this paper has several advantages over the AMT recommended by Wardle et al. [36,37]: (1) no prior knowledge of the bifurcation modes is needed, and (2) the same mesh is used for tracing all secondary paths of the structure. The accuracy and reliability of this method is tested on the benchmark example.

This paper is organized as follows. In Section 2, we briefly introduce the nonlinear buckling analysis algorithm. In Section 3, we apply the method to a benchmark example and compare with the results available in the literature. Additional bifurcation points and secondary paths are also obtained in this section. In Section 4, we perform a parametric study of the influence of the rises, thicknesses and boundary conditions on the variation of critical points and postbuckling responses. Conclusions are outlined in Section 5.

2. Nonlinear buckling and postbuckling analysis

In this section, we briefly introduce a numerical procedure, combining the arclength and branch-switching methods, which can reliably determine all critical points and corresponding postbuckling responses including bifurcated secondary paths.

2.1. Critical points on the equilibrium path

An elastic system typically loses stability when the tangent stiffness $\mathbf{K}$ becomes singular. Points on the equilibrium path with singular tangent stiffness are called critical points, further differentiated as limit and bifurcation points (Fig. 1). A null right eigenvector $\mathbf{z}$ of the tangent stiffness $\mathbf{K}$ at a critical point satisfies

$$\mathbf{K}\mathbf{z} = \mathbf{0}. \tag{1}$$

When an elastic structure is subjected to a conservative loading, the tangent stiffness $\mathbf{K}$ is symmetric and Eq. (1) also implies $\mathbf{z}^T\mathbf{K} = \mathbf{0}$. For an incremental-iterative method, the incremental displacement $\Delta\mathbf{u}$ and loading $\Delta\lambda$ satisfy $\mathbf{K}\Delta\mathbf{u} = \Delta\lambda\mathbf{q}$. Premultiplying both sides with $\mathbf{z}^T$ and using $\mathbf{z}^T\mathbf{K} = \mathbf{0}$, we get

$$\mathbf{z}^T\mathbf{q}\Delta\lambda = 0 \tag{2}$$

Three configurations satisfy Eq. (2): (1) $\Delta\lambda = 0$, denoting a limit point (Fig. 1(a)); (2) $\mathbf{z}^T\mathbf{q} = 0$, indicating a bifurcation point (Fig. 1(b)); or (3) $\Delta\lambda = 0$ and $\mathbf{z}^T\mathbf{q} = 0$ simultaneously, implying the coincidence of a bifurcation and limit point. In practice, limit points are indeed identified as points of zero variation in the load factor, but bifurcation points are not detected based on the above. Instead in this paper, several lowest eigenvalues of the tangent stiffness $\mathbf{K}$ are monitored when tracing the primary equilibrium path. Zero eigenvalues of the tangent stiffness indicate the location of critical points, out of which, those not already identified by $\Delta\lambda = 0$ are the bifurcation points. For the case of a multiple bifurcation point or of the coincidence of a limit point and a bifurcation point, multiple eigenvalues are zero at the same time. Finally, note that only conservative systems are considered in this paper.

2.2. Switching to secondary paths

After the detection of bifurcation points, the branch-switching method proposed in [38,39] is adopted to switch from the primary equilibrium path to a secondary path. At a simple bifurcation point, the eigenvector $\boldsymbol{\phi}_j$ of the zero eigenvalue λ_j indicates the direction of one secondary path $\mathbf{j}$, and can be used as a perturbation of the solution on the primary path. To switch to the secondary path $\mathbf{j}$, the eigenvector $\boldsymbol{\phi}_j$ is scaled and added to the solution in the following way:

$$\mathbf{u}_j = \mathbf{u} \pm \frac{\|\mathbf{u}\|}{\tau_j}\frac{\boldsymbol{\phi}_j}{\|\boldsymbol{\phi}_j\|} \tag{3}$$

where τ_j is a scaling factor, $\mathbf{u}$ is the converged displacement vector on the primary path, and $\mathbf{u}_j$ represents a predictor for the secondary path $\mathbf{j}$. The arclength method can then be used to correct the predictor $\mathbf{u}_j$ and follow additional solutions on the secondary path $\mathbf{j}$.

Two important aspects of this branch-switching method are noted here. First, two directions are typically associated with one secondary path, as shown in Fig. 1(b) and they correspond to the plus and minus sign in Eq. (3). Second, the value of the scaling factor τ_j is usually less than 100 based on our simulation experience (a too large value can lead to a solution that remains on the primary path, while a too small one may lead to divergence). An adaptive approach with a restart option that can rerun a new simulation directly from the bifurcation point is therefore recommended to

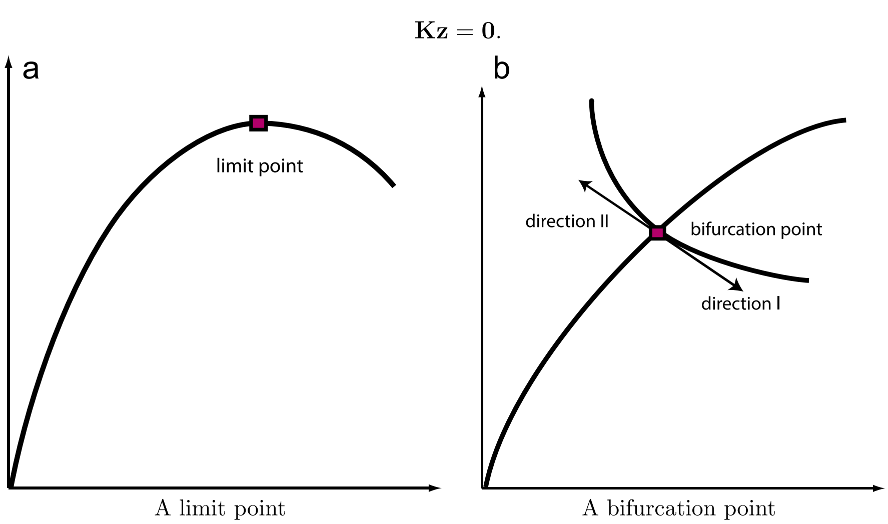

Fig. 1. Critical points on equilibrium paths. (a) A limit point. (b) A bifurcation point.