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ScienceDirect
Finite Elements in Analysis and Design 43 (2007) 1013–1022
FINITE ELEMENTS IN ANALYSIS AND DESIGN
www.elsevier.com/locate/finel

Free vibration and buckling analyses of composite plates with straight-sided quadrilateral domain based on DSC approach

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Received 13 January 2007; received in revised form 24 June 2007; accepted 27 June 2007
Available online 26 July 2007

Abstract

Discrete singular convolution (DSC) method has been proposed to obtain the frequencies and buckling loads of composite plates. By using geometric transformation, the straight-sided quadrilateral domain is mapped into a square domain in the computational space using a four-node element. Plates having different geometries such as rectangular, skew, trapezoidal and rhombic plates are presented. The obtained results are compared with those of other numerical methods. Numerical results indicate that the DSC is a simple, accurate and reliable algorithm for vibration and buckling analyses of composite plates.

Keywords: Discrete singular convolution; Composite; Skew plate; Trapezoidal plate; Free vibration; Buckling

1. Introduction

A variety of numerical methods are available today for engineering analysis. These numerical approaches have been used extensively for solving linear and nonlinear differential equations. Discrete singular convolution (DSC) method is a new method that was introduced by Wei [1,2]. As stated by Wei [3–5], singular convolutions are a special class of mathematical transformations, which appear in many science and engineering problems, such as the Hilbert, Abel and Radon transforms. In fact, these transforms are essential to many practical applications, such as computational electromagnetic signal and image processing, pattern recognition, topography, molecular potential surface generation and dynamic simulation [6,7]. Several researchers have applied the DSC method to solve a variety of problems in different fields of science and engineering [8–15].

The analysis of composite plates and straight-sided plates has been the subject of the research of structural and mechanical engineering [16–28]. Li et al. [22] presented a spline finite strip analysis of arbitrary-shaped general plates. Cheung et al. [23] also developed a finite strip analysis for static and vibration analysis of general plates. In these studies, cubic serendipity shape functions were first employed for arbitrary-shaped general plates by finite strip method. Following Wang and Cheng, Lim et al. [28] also used a similar approach to analyze irregular plates using the finite strip method in conjunction with orthogonal polynomials. Kitipornchai et al. [29] studied the buckling of skew plates by the method of Rayleigh–Ritz method. Liew and Han [25,30] introduced a mapping technique to apply the differential quadrature (DQ) method for analysis of plates. Buckling analysis of skew plates has been presented by Wang et al. [31]. Liew et al. [32,33] have investigated vibration characteristic of thick skew plates. Chen et al. [34] investigated the shear deformation for free vibration of symmetrically laminated thick-trapezoidal plates using p-Ritz method. Three-dimensional vibration analysis of a cantilevered parallelepiped is presented by Lim [35] via exact and approximate solutions.

Composite laminated plates are also common structural elements in many kinds of high-performance surface and air vehicles. Such structures are widely used reinforced slabs or plates, ship hulls, as floors in bridges, fiber reinforced plastic structures. Thus, frequencies and buckling loads of such structures are important in the design of systems. As a consequence, the vibration and buckling of isotropic and laminated composite plates have been extensively studied [26–28,36–39]. A long list of references on free vibration of laminated plates are given, for example, in Refs. [40–43]. In the present study, free vibration

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doi:10.1016/j.finel.2007.06.014

and buckling analysis of composite plate is examined by the method of DSC. The given results are verified by comparison against results available in the open literature. To the author's knowledge, this is the first instance in which the DSC method has been adopted for free vibration and buckling analysis of composite plates.

2. Discrete singular convolution

The DSC method is an efficient and useful approach for the numerical solutions of differential equations. This method was introduced by Wei [1] in 1999. Like some other numerical methods, the DSC method discretizes the spatial derivatives and, therefore, reduces the given partial differential equations into a standard eigenvalue problem. The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis. Wei and his co-workers first applied the DSC algorithm to solve solid and fluid mechanics problems [10–14]. Zhao et al. [6,8,9] analyzed the high-frequency vibration of structures using DSC algorithm. Civalek [44–46] gives numerical solution of free vibration problem of rotating and laminated conical shells, plates on elastic foundation. These studies indicate that the DSC algorithm works very well for the vibration analysis of plates, especially for high-frequency analysis of rectangular plates. More recently, Lim et al. [14,15] presented the DSC–Ritz method for the free vibration analysis of Mindlin plates and thick shallow shells.

Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [7]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)\,\mathrm{d}x, \tag{1}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type

$$T(x) = \delta^{(n)}(x) \quad (n = 0, 1, 2, \ldots). \tag{2}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n > 1$ are essential for numerically solving differential equations. The Shannon's kernel is regularized as [8]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0, \tag{3}$$

where Δ is the grid spacing. Eq. (3) can also be used to provide discrete approximations to the singular convolution kernels of the delta type [10]:

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_\Delta(x - x_k) f(x_k), \tag{4}$$

where $\delta_\Delta(x-x_k) = \Delta\delta_\alpha(x-x_k)$ and superscript (n) denotes the nth-order derivative, and $2M+1$ is the computational bandwidth which is centered around x and is usually smaller than the whole computational domain.

In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x - x_M, x + x_M]$.

3. DSC method for irregular domains

3.1. Straight-sided quadrilateral plates

Consider an arbitrary straight-sided quadrilateral plate in the Cartesian x–y plane, as shown in Fig. 1(a). The geometry of this plate can be mapped into a rectangular plate in the natural ξ–η plane, as shown in Fig. 1(b). By employing the following transformation equations, the physical domain is mapped into the computational domain:

$$x = \sum_{i=1}^{N} x_i \Phi_i(\xi, \eta) \tag{5}$$

and

$$y = \sum_{i=1}^{N} y_i \Phi_i(\xi, \eta), \tag{6}$$

where x_i and y_i are the coordinates of node i in the physical domain, N is the number of grid points, and $\Phi_i(\xi, \eta)$; $i = 1, 2, 3, \ldots, N$ are the interpolation or shape functions. These are given for node i:

$$\Phi_i(\xi, \eta) = \tfrac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i). \tag{7}$$

Using the chain rule, the first-order, and second-order derivatives of a function are given.

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = [J_{11}]^{-1} \begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix}, \tag{8}$$

$$\begin{Bmatrix} u_{xx} \\ u_{yy} \\ 2u_{yx} \end{Bmatrix} = [J_{22}]^{-1} \begin{Bmatrix} u_{\xi\xi} \\ u_{\eta\eta} \\ 2u_{\xi\eta} \end{Bmatrix} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix}, \tag{9}$$

where ξ_i and η_i are the coordinates of node i in the ξ–η plane, and J_{ij} are the elements of the Jacobian matrix. These are expressed as follows:

$$[J_{11}] = \begin{bmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{bmatrix}, \tag{10}$$

$$[J_{21}] = \begin{bmatrix} x_{\xi\xi} & y_{\xi\xi} \\ x_{\eta\eta} & y_{\eta\eta} \\ x_{\xi\eta} & y_{\xi\eta} \end{bmatrix}, \tag{11}$$

$$[J_{22}] = \begin{bmatrix} x_\xi^2 & y_\xi^2 & x_\xi y_\xi \\ x_\eta^2 & y_\eta^2 & x_\eta y_\eta \\ x_\xi x_\eta & y_\xi y_\eta & \frac{1}{2}(x_\xi y_\eta + x_\eta y_\xi) \end{bmatrix}. \tag{12}$$

The above transformations will be used later to transform the governing differential equations and related boundary conditions from the physical domain x–y into the computational domain ξ–η. Thus, an arbitrary-shaped quadrilateral plate may be