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Feature sensitivity: A generalization of topological sensitivity

Sankara Hari Gopalakrishnan, Krishnan Suresh*

Department of Mechanical Engineering, University of Wisconsin, 3108 Engineering Centers Building, 1550 Engineering Drive, Madison, WI 53706, USA

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ABSTRACT

Shape and topology optimization have flourished over the last two decades, resulting in a number of powerful mathematical concepts. One such concept is that of *topological sensitivity* that quantifies the impact of adding *infinitesimal* holes (within a given continuum) on specific quantities of interest such as compliance, average stress, etc. In this paper we explore a novel generalization of topological sensitivity called *feature sensitivity* that captures the first-order change in quantities of interest when an arbitrary internal and boundary feature is created within an existing continuum. Specific algorithms are derived for computing the feature sensitivity of linear elasticity problems, and illustrated through numerical experiments.

1. Introduction

Engineering products are routinely designed and analyzed today in a virtual CAD/CAE environment, with minimal reliance on expensive prototyping. Engineering analysis methods such as the finite element, boundary-element, finite-difference and finite volume methods [1–5] have reached a high degree of reliability. In addition, shape and topology optimization methods [6–8] are being used in conjunction with the analysis methods, to maximize the performance of designs, while satisfying typical engineering constraints. Among the many mathematical concepts that underlie shape and topology optimization [8–10], we review below two concepts particularly relevant to this paper.

Consider a 2-D geometry that is subject to certain external forces and constraints as shown in Fig. 1. The boundary value problem can be easily solved via (say) finite element analysis (FEM) [5]. Consequently, one can compute various *quantities of interest* such as system compliance, overall potential energy, average stress within a region, etc.

The primary concept employed in shape optimization is that of *shape sensitivity* that addresses the question on how these quantities of interest would change when the *boundary* is perturbed *infinitesimally*; see Fig. 2a. Various methods have been proposed for computing shape sensitivity; see [9].

Topology optimization, on the other hand, can be achieved either via a material approach or a geometric approach [6]. A particular geometric approach, that is gaining popularity, is based on the notion of *topological sensitivity*. The latter complements shape sensitivity by addressing the question on what would happen to the quantities of interest if an *infinitesimal hole* is added to the domain; see Fig. 2b. Note that since topological sensitivity depends on where the hole is added, one obtains a topological sensitivity *field* defined everywhere within the domain. Various methods have been proposed to compute the topological sensitivity, for example, see [11–17]. Furthermore, it has been demonstrated in [12] that the notion of topological sensitivity can be extended to the boundary in that one can legitimately find the sensitivity of placing holes on the boundary (without changing topology), making it a very attractive mathematical concept.

Now consider the following question: *What would be the effect on a quantity of interest when a finite-size region/feature of arbitrary shape and size is subtracted from the design?*[1]

The above question is of significant importance in mechanical design for the following reasons:

1. Topological sensitivity, in theory, applies only to infinitesimal holes, while, in practice, engineers are often interested in creating finite-size modifications to existing designs. Thus, determining the sensitivity of a design to finite feature deletions could be of immense value to the designer.
2. In detail removal (a.k.a. defeaturing) one typically deletes 'small' or 'irrelevant' features prior to simulation [18–20]. Quantifying the effect of defeaturing on down-stream simulation is still an open challenge, and is directly related to the posed question.

* Corresponding author. Tel.: +1 608 262 3594; fax: +1 608 262 2316.
E-mail address: suresh@engr.wisc.edu (K. Suresh).

[1] In this paper, 'feature' refers to any sub-set of the geometry, and may overlap with the boundary; feature subtraction or feature deletion refers to the removal of this sub-set from the base geometry.

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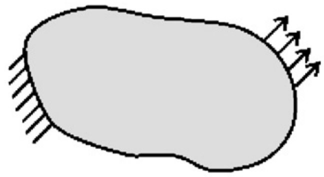

Fig. 1. A structural boundary value problem.

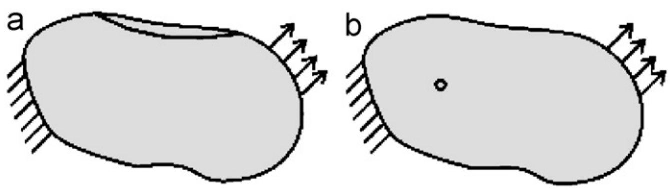

Fig. 2. (a) Shape sensitivity and (b) topological sensitivity.

3. Finally, recognizing that most modern CAD systems are feature-based, the ability to directly measure the sensitivity of feature subtraction during CAE analysis can greatly enhance CAD/CAE coupling. Indeed, a recent NSF report [21] identifies 'lack of CAD/CAE integration' as one of the most critical issues in product development.

Towards addressing this question, we introduce the concept of feature sensitivity that captures the first-order change in quantities of interest when a cluster of small internal and boundary features of arbitrary-shape is created within the geometry.

Feature sensitivity may be viewed as a generalization of the topological sensitivity concept and can be computed via a robust post-processing step, and can therefore be executed rapidly. The numerical experiments rely on standard FEA [3–5] and boundary element analysis (BEA) [1] for much of the underlying computation. The methodology has been implemented within COMSOL [22], a commercially available finite-element based CAD/CAE system. Further, we show through numerical experiments that the proposed method can be highly accurate.

2. Problem statement

Consider a 2-D base-design Ω in Fig. 3 over which we pose the following standard linear elasticity problem [23]:

$$\begin{aligned} &-\nabla\cdot\boldsymbol{S}_0=\boldsymbol{f} \quad \text{in } \Omega\\ &\boldsymbol{u}_0=\hat{\boldsymbol{u}} \quad \text{on } \partial\Omega_D\\ &\boldsymbol{S}_0\boldsymbol{n}=\boldsymbol{q} \quad \text{on } \partial\Omega_N \end{aligned} \tag{2.1}$$

where $\boldsymbol{S}_0=[\![\boldsymbol{C}]\!]\Delta\boldsymbol{u}_0$

$$\Delta\boldsymbol{u}_0 \equiv \tfrac{1}{2}(\nabla\boldsymbol{u}_0+\nabla\boldsymbol{u}_0^{\mathrm{T}})$$

where $\boldsymbol{u}_0$ is the displacement field, etc. [23]. We shall assume that the above problem over the base-design in Fig. 3 has been solved, via, say FEA.

Further, we shall assume that one has computed a generic quantity of interest φ defined per:

$$\varphi_0=\int_R g(\boldsymbol{u}_0,\nabla\boldsymbol{u}_0)\,\mathrm{d}\Omega \tag{2.2}$$

where g is some non-linear function, defined over a region of interest $R\subset\Omega-\omega$ (see Fig. 4).

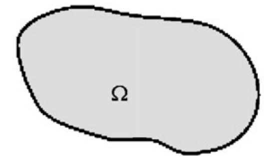

Fig. 3. Base-design.

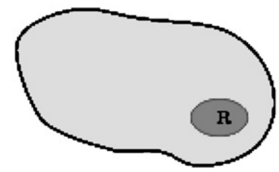

Fig. 4. Region of interest.

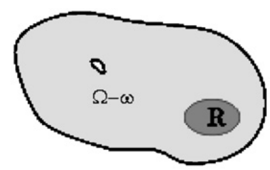

Fig. 5. Modified design.

2.1. Internal features

Now consider the topologically modified design in Fig. 5 where a small internal feature has been inserted. To facilitate analysis, we shall represent this geometry as $\Omega-\omega$, where ω is the small internal feature.

We now pose a modified linear elasticity problem (analogous to Eq. (2.1)):

$$\begin{aligned} &-\nabla\cdot\boldsymbol{S}=\boldsymbol{f} \quad \text{in } \Omega-\omega\\ &\boldsymbol{u}=\hat{\boldsymbol{u}} \quad \text{on } \partial\Omega_D\\ &\boldsymbol{S}\boldsymbol{n}=\boldsymbol{q} \quad \text{on } \partial\Omega_N\\ &\boldsymbol{S}\boldsymbol{n}=\boldsymbol{0} \quad \text{on } \partial\omega \end{aligned} \tag{2.3}$$

where $\boldsymbol{S}=[\![\boldsymbol{C}]\!]\Delta\boldsymbol{u}$

$$\Delta\boldsymbol{u} \equiv \tfrac{1}{2}(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{\mathrm{T}})$$

Observe that zero Neumann conditions are imposed on the inserted feature boundary; other boundary conditions can be handled as well. As before, we also define a quantity of interest φ (analogous to Eq. (2.2)):

$$\varphi=\int_R g(\boldsymbol{u},\nabla\boldsymbol{u})\,\mathrm{d}\Omega \tag{2.4}$$

The problem addressed in this paper may be summarized as follows:

Given the solutions to Eqs. (2.1) *and* (2.2), *estimate* φ *in Eq.* (2.4) *without solving Eq.* (2.3).

2.2. Boundary features

In the previous section we modified the topology by inserting an internal feature. While internal features are fairly common in 2-D, they are very rare in 3-D, i.e., most features in 3-D intersect the boundary. For example, Fig. 6 illustrates a boundary feature. The problem now is analogous to the one described above, i.e., we would