Finite Elements in Analysis and Design 86 (2014) 61–70

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Integrating a logarithmic-strain based hyperelastic formulation into a three-field mixed finite element formulation to deal with incompressibility in finite-strain elastoplasticity

Dina Al Akhrass [a,b,c,*], Julien Bruchon [a], Sylvain Drapier [a], Sébastien Fayolle [b,c]

[a] Ecole des Mines de Saint-Etienne, Centre SMS, LGF UMR CNRS 5307, 158 cours Fauriel, 42023 Saint-Etienne Cedex 2, France
[b] EDF R&D, Département AMA, 1, Avenue du Général de Gaulle, 92141 Clamart, France
[c] Laboratoire de Mécanique des Structures Industrielles Durables, UMR EDF-CNRS-CEA 8193, 1, Avenue du Général de Gaulle, 92141 Clamart, France

ARTICLE INFO

Article history:
Received 6 September 2013
Received in revised form
20 February 2014
Accepted 1 April 2014
Available online 28 April 2014

Keywords:
Incompressibility
Finite-strain
Mixed finite element method
Logarithmic strains

ABSTRACT

This paper deals with the treatment of incompressibility in solid mechanics in finite-strain elastoplasticity. A finite-strain model proposed by Miehe, Apel and Lambrecht, which is based on a logarithmic strain measure and its work-conjugate stress tensor is chosen. Its main interest is that it allows for the adoption of standard constitutive models established in a small-strain framework. This model is extended to take into account the plastic incompressibility constraint intrinsically. In that purpose, an extension of this model to a three-field mixed finite element formulation is proposed, involving displacements, a strain variable and pressure as nodal variables with respect to standard finite element. Numerical examples of finite-strain problems are presented to assess the performance of the formulation. To conclude, an industrial case for which the classical under-integrated elements fail is considered.

1. Introduction

Over the past three decades, the development of finite element formulations capable of modeling large elastoplastic strains has received considerable attention. The earliest formulations, introduced in the 1970s, were rate-type formulations based on a hypoelastic stress–strain relationship [1–3]. The main drawback of this kind of formulations is that they introduce elastic dissipation. Despite that, they are still commonly used today. In the 1980s, Simo and co-workers [4–7] introduced a formulation based on hyperelastic constitutive formulations that does not exhibit elastic dissipation. Many researchers, such as Eterovic and Bathe [8] or Weber and Anand [9], developed some formulations that use a hyperelastic constitutive model based on the logarithmic strain tensor, also called the Hencky tensor. In these works, an additive decomposition of the elastic and plastic strains in the absence of rotations is assumed, which is a typical feature of the geometrically linear theory of plasticity. This provides a natural basis for a material-independent extension of constitutive structures from the geometrically linear to the nonlinear theory at finite-strain. More recently, Miehe et al. [10] developed a formulation for which the kinematic setting consists of a constitutive model in the logarithmic strain space that is preceded and followed by purely geometric processing. In such a way, the relationship between the large-strain and the small-strain setting is defined by purely geometric transformations. When modeling finite-strain elastoplastic processes by the finite element method, it is important to consider the nearly incompressible plastic behavior, which is typical of metals, for instance. It is well-known that the standard displacement-based finite element formulation performs poorly in quasi-incompressible situations, producing stiff solutions and oscillations of the stress field. Over the years, and particularly in the 1990s, different strategies were proposed to reduce or avoid volumetric locking and pressure oscillations in finite element solutions. Several methods to deal with incompressibility have been developed, such as under-integrated elements [11], Enhanced Assumed Strain (EAS) methods [12–14], B-bar and F-bar methods [15,16], or mixed formulations [17,18]. The mixed finite element method is robust and generic and it is a popular and efficient way to deal with incompressibility. Mixed elements for finite strains were first introduced by Simo et al. [19] and have been since developed by many authors [20,21]. In non-linear solid mechanics, the use of a two-field formulation is not convenient for many constitutive models [20,22]. For example when the plasticity criterion depends on the hydrostatic stress like in Rousselier or GTN laws [23,24], a two-field formulation based on displacement and pressure is not straightforward and differ from the chosen law. For this kind of laws, the use of a three-field formulation in which the unknowns are the displacement, the pressure and the volumetric strain fields, allows us to have a generic method. Furthermore, in the finite-

* Corresponding author. Tel.: +33 6 11 98 75 89.
E-mail address: alakhrass@emse.fr (D. Al Akhrass).

http://dx.doi.org/10.1016/j.finel.2014.04.004
0168-874X/

strain framework, it is important to express these formulations in a way that makes the implementation of the constitutive laws simple. That is why the idea in this work is to adopt the model of Miehe et al. [10], and to adapt it to a three-field formulation.

This paper presents a robust non-linear mixed finite element procedure for the numerical analysis of finite-strain elastoplasticity, where the use of specific elements is considered to deal with plastic incompressibility. The Miehe's model, based on a logarithmic description of the strain tensor, is exposed in Section 2. A three-field mixed finite element formulation is then presented in Section 3. The extension of the finite-strain model to this three-field formulation is described in Section 4. Finally, some numerical simulations are presented in Section 5. The efficiency of the developed model is first shown through numerical tests. A case of industrial interest, for which under-integration technique fails, is then considered in order to assert the robustness of the presented approach.

2. Description of the finite-strain model

In order to describe the finite-strain framework, a hyperelastic-based model developed by Miehe, Apel and Lambrecht is considered [10,25]. The kinematic framework consists of a constitutive model in the logarithmic strain space that is framed by purely geometric pre-processing and post-processing steps. This numerical approach of the material response computation can be split into three steps: a geometric pre-processing step in which the logarithmic strain tensor is defined, a second step to get its work-conjugate stresses computed from the constitutive model, and a third one to get back to classical tensors using geometric post-processing. Note that in the present work, Lagrangian tensors are considered.

2.1. Geometric pre-processing

This step consists in defining the logarithmic strain tensor, and the geometric transformations necessary to get it along with its associated stress tensor, from standard tensors. The logarithmic strain tensor $\boldsymbol{E}$ is defined by

$$\boldsymbol{E} = \frac{1}{2}\ln(\boldsymbol{C}) \tag{1}$$

with $\boldsymbol{C}$ the right Cauchy–Green strain tensor,

$$\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F} \tag{2}$$

where $\boldsymbol{F}$ is the deformation gradient tensor defined as the relative deformation of the medium from its initial state (position $\boldsymbol{X}$) to its current state (position $\boldsymbol{x}$)

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \tag{3}$$

Following Miehe's approach [10], an additive decomposition of the logarithmic strain is considered

$$\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^p \tag{4}$$

where $\boldsymbol{E}^e$ is referred as the elastic strain, and $\boldsymbol{E}^p$ as the plastic strain. It has been shown in Refs. [10,25,26] that this additive decomposition provides results close to those obtained by assuming Lee's multiplicative decomposition of the deformation gradient [27].

$\boldsymbol{E}^p$ is assumed to be a function of the plastic metric tensors $\boldsymbol{G}^p$ [26] defined by

$$\boldsymbol{G}^p = \boldsymbol{F}^{pT}\boldsymbol{F}^p \tag{5}$$

and can be expressed as

$$\boldsymbol{E}^p = \frac{1}{2}\ln(\boldsymbol{G}^p) \tag{6}$$

Thus, the logarithmic strain tensor allows us to switch from multiplicative properties for the elastoplasticity in finite strain to the additive structure of the small strain theory. Furthermore, the plastic Jacobian denoted J^p is such that

$$J^p := \sqrt{\det(\boldsymbol{G}^p)} = \exp[\mathrm{tr}(\boldsymbol{E}^p)] \tag{7}$$

which gives

$$\mathrm{tr}(\boldsymbol{E}^p) = \ln(J^p) \tag{8}$$

Thus, it can be written that

$$\det(\boldsymbol{G}^p) = 1 \Leftrightarrow \mathrm{tr}(\boldsymbol{E}^p) = 0 \tag{9}$$

Indeed, the multiplicative constraint on the determinant of the plastic metric is described by the additive constraint on the trace of the logarithmic plastic strain.

The model is based on the logarithmic strain tensor (1) and its work-conjugate stress tensor denoted $\boldsymbol{T}$. In order to get the expression of $\boldsymbol{T}$, in terms of standard tensors, the expended power is expressed as a function of the second Piola–Kirchhoff stress tensor $\boldsymbol{S}$ and the right Cauchy–Green strain tensor $\boldsymbol{C}$

$$\mathcal{P}(t) = \boldsymbol{S}(t) : \frac{1}{2}\dot{\boldsymbol{C}}(t) \tag{10}$$

with

$$\dot{\boldsymbol{C}}(t) = \frac{\partial \boldsymbol{C}}{\partial t} \tag{11}$$

and as a function of $\boldsymbol{E}$ and $\boldsymbol{T}$

$$\mathcal{P}(t) = \boldsymbol{T}(t) : \dot{\boldsymbol{E}}(t) \tag{12}$$

so that $\boldsymbol{T}$ can be expressed as [10]

$$\boldsymbol{T} = \boldsymbol{S} : \boldsymbol{P}_L^{-1} \tag{13}$$

with

$$\boldsymbol{P}_L = 2\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{C}} \tag{14}$$

Thus, geometric relationships between the logarithmic strain tensor $\boldsymbol{E}$ and its work-conjugate stress tensor $\boldsymbol{T}$ with respectively, the right Cauchy–Green strain tensor, and the second Piola–Kirchhoff stress tensor, have been established.

2.2. Constitutive model in the logarithmic strain space

Let us assume a constitutive model of plasticity that is written in the logarithmic strain space. From the logarithmic strain tensor $\boldsymbol{E}$ and the logarithmic plastic strain tensor $\boldsymbol{E}^p$, calculated respectively by (1) and (6), and some hardening variables denoted α, the constitutive model provides the stress tensor $\boldsymbol{T}$ and the associated elastoplastic tangent modulus $\mathbb{E}^{ep}$

$$\{\boldsymbol{E}, \boldsymbol{E}^p, \alpha\} \Rightarrow \text{Constitutive model} \Rightarrow \{\boldsymbol{T}, \mathbb{E}^{ep}\} \tag{15}$$

The tangent modulus yields the rate of the stress $\boldsymbol{T}$ with respect to the rate of the logarithmic strain

$$\dot{\boldsymbol{T}} = \mathbb{E}^{ep} : \dot{\boldsymbol{E}} \tag{16}$$

Note that, in the logarithmic strain space, the constitutive model has the same structure as models for small-strain plasticity. It is hence possible to adopt simply, for finite-strain elastoplasticity, the frame of constitutive models from the small-strain theory.