Finite Elements in Analysis and Design 86 (2014) 81–88

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Efficient computation of wave propagation along axisymmetric pipes under non-axisymmetric loading

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ARTICLE INFO

Article history:
Received 6 November 2013
Received in revised form
19 March 2014
Accepted 1 April 2014
Available online 4 May 2014

Keywords:
Non-axisymmetric loading
Spectral element
Axisymmetric geometry

ABSTRACT

This paper presents an efficient formulation of the problem of wave propagation along the length of axisymmetric pipes under non-axisymmetric loading such as leaks or new cracks so that wave characteristics in a pipe can be identified without the excessive computational time associated with most current 3D modeling techniques. The axisymmetric geometry of the pipe is simplified by reducing the problem to 2D while the non-axisymmetric loading is represented by the summation of Fourier series. Since the pipe stiffness matrix as conventionally formulated represents the greatest single computational load, the strain–displacement matrix is partitioned in such a way that numerical integration components are decoupled from θ (the angular parameter) and n (the number of Fourier terms). A single numerical integration of the strain–displacement matrix is performed and utilized for all the iterations of Fourier terms to represent the non-axisymmetric load. The numerical formulation is conducted using spectral elements, which also reduce computational time since these elements yield a diagonal mass matrix. The computational efficiency of the developed method is compared with conventional finite element tools.

1. Introduction

The main wave propagation based Structural Health Monitoring (SHM) methods for pipelines are guided wave ultrasonics and acoustic emission. Guided wave ultrasonics relies on capturing the reflected wave energy from a defect after introducing a perturbation signal using ultrasonic transducers [1,2]. If less-dispersive guided modes are selected to transmit and receive the signal, long-range pipes can be monitored using a limited set of transducers. The acoustic emission (AE) method relies on propagating elastic waves emitted from newly formed damage surfaces such as active cracks and leaks. Crack growth causes sudden stress–strain change in its vicinity, which generates a wideband step function. A leak causes turbulence at its location, which generates continuous emissions. The AE method may be based on elastic waves propagating through the pipe material [3,4] or acoustic waves propagating through the material inside the pipe [5]. For an effective and accurate monitoring approach, wave characteristics such as the dispersion curves under buried or fluid filled conditions and the attenuation profile should be known prior to the implementation of an SHM method. However, experimental simulations of different pipe geometries and conditions are generally not possible. Wave propagation in pipes is a complex phenomenon due to the excitation of multi-mode waves, which must be superimposed to provide an overall solution. Analytical solutions of governing differential equations are not applicable when the pipe geometry becomes complex with the presence of defects, coatings and internal materials [6,7], buried conditions [8], and pipe bends [9]. The modeling of wave propagation is important for quantitative understanding of damage mechanics and the identification of the SHM system characteristics (e.g. frequency selection, sensor position [10]).

Wave propagation in pipes can be numerically modeled as 2D or 3D [11]. The 3D wave propagation problems of hollow circular cylinders including non-axisymmetric wave modes are formulated by Gazis [12]. If the problem requires modeling high frequency waves in a large-scale structure, the 3D model becomes computationally expensive. Therefore, it is imperative to reduce the mathematical problem to 2D or implement semi-analytical finite element formulation [13,14] for reducing the computational load. When the structure and loading are axisymmetric, the structural model can be reduced to a 2D problem as displacements and stresses are independent of θ (the angular parameter). There are several other methods for reducing the computational time of high frequency wave propagation in hollow structures. Benmeddour et al. [15] developed a three dimensional hybrid method which combined a classical FE method and normal mode expansion technique in order to study the interaction of guided waves with

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http://dx.doi.org/10.1016/j.finel.2014.04.001
0168-874X/

non-axisymmetric cracks in cylinders. Zhou et al. [16] utilized the numerical eigenmode extraction method applicable to wave propagation in periodic structures. Mazzotti et al. [17] applied a Semi Analytical Finite Element (SAFE) method to study the influence of prestressing load on the dispersion of viscoelastic pressurized pipe. Bai et al. [14] constructed an elastodynamic steady-state Green's function based on modal data determined from the spectral decomposition of a circular laminated piezoelectric cylinder using semi-analytical finite element formulation. Zhuang et al. [13] proposed integral transform and forced vibration as two different methods both based on the same set of eigenvalue data to construct a steady-state Green's function for laminated circular cylinder. Gsell et al. [18] discretized the displacement equations directly using the finite difference method, which reduced the computational time by 25%.

When the structure is axisymmetric and the load is non-axisymmetric, three displacement components in the radial, axial and circumferential directions exist [19,20]. To utilize the 2D axisymmetric geometry of the pipe, the non-axisymmetric load can be expanded using Fourier series, and the structural response can be computed by superposing the solutions of each Fourier term [21,22]. There are several examples in the literature related to applying Fourier series summation to model axisymmetric geometries with non-axisymmetric loading such as Zhuang et al. [13], Bai et al. [14],Wunderlich et al. [23] and Bouzid et al. [24]. However, in case where the Fourier expansion of the load function requires many harmonics to represent the non-axisymmetric loads such as concentrated loads, the 2D superposition method combined with conventional finite element formulation may not be computationally efficient than 3D analyses [25,26]. Bathe [25] describes that the stiffness matrices corresponding to the different harmonics can be decoupled due to the orthogonal properties of trigonometric functions. However, to the best of our knowledge, there is no study that explicitly presents the mathematical formulation. While numerical models are capable of simulating various pipe geometries and conditions to deduce the waveform characteristics, existing numerical formulations for wave propagation are computationally expensive, and not practical for modeling long-range pipes.

In this paper, a detailed mathematical formulation is presented to model axisymmetric geometries with non-axisymmetric dynamic loads, specifically concentrated loads. The formulation is based on partitioning the strain–displacement matrix in such a way that the numerical integration terms are decoupled from the variables θ and n (the number of Fourier terms). Therefore, the strain–displacement matrix is calculated only once and used for each Fourier term calculation. Additionally, the numerical formulation is built using spectral elements, which are special forms of finite elements that yield a diagonal mass matrix and thereby provide highly efficient numerical models for high frequency wave propagation [27,28,29]. The nodal coordinates of the Lagrange shape functions are obtained from the solutions of orthogonal polynomials. Moreover, one could optimize the computational expenditure by expanding the displacement field in the pipe over a series of normal modes.

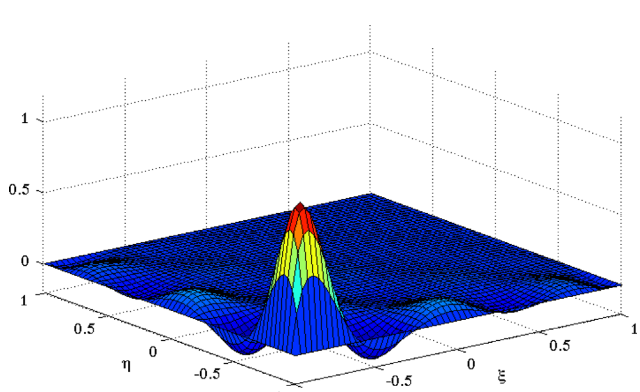

Fig. 1. Lagrangian 2D shape function using GLL integration points for the coordinate of (0.87, 0.87).

The organization of this article is as follows: the discretization, spectral element formulation and the non-axisymmetric loading formulation are described in Section 2. In Section 3, the 2D numerical results are compared with 3D finite element models. The computational efficiency of the 2D model is presented in Section 4. Finally, the major outcome of this study and future work are summarized in Section 5.

2. Method of analysis

2.1. Discretization

For the spectral element formulation, the discrete locations of the nodal coordinates are defined by the Gauss–Lobatto–Legendre (GLL) points using the selected Legendre polynomial degree, which defines the p refinement within the element. The GLL points are calculated by the roots of the following equation [30]:

$$(1-\xi^2)\frac{dP_N(\xi)}{d\xi}=0 \tag{1}$$

where $P_N(\xi)$ is the Legendre polynomial of degree N and $\xi\in[-1,1]$. The basis function $N_{ij}\,(=0,\ldots,\text{number of node})$ in 2D is expressed using the Lagrange interpolation polynomials [31]

$$N_{ij}=\prod_{\substack{m=0\\m\neq i}}^{n}\left(\frac{\xi-\overline{\xi}_m}{\overline{\xi}_i-\overline{\xi}_m}\right)\prod_{\substack{l=0\\l\neq j}}^{n}\left(\frac{\eta-\overline{\eta}_l}{\overline{\eta}_j-\overline{\eta}_l}\right) \tag{2}$$

where ξ and η are the natural coordinate systems in the element and $\overline{\xi}_i$ and $\overline{\eta}_j$ are the coordinates of the ith and jth node in the directions of ξ and η, respectively. Fig. 1 shows the Lagrangian shape function using GLL integration points for (ξ,η). The non-uniform distribution of nodal points reduces the presence of Runge phenomenon [32].

2.2. Coordinate system and corresponding displacement fields

The cylindrical coordinates and the axis of revolution for pipe geometry are shown in Fig. 2(a). The axial, radial and circumferential displacements are defined as w, u and v, respectively. The spectral element discretization of a cross sectional element using the 5th order Legendre polynomial is shown in Fig. 2b. The axis of revolution is z with the displacement component w.

In general, when loading has no symmetry and is defined by Fourier series components, the displacement components at radial (u, axial (w and circumferential (v) directions can be defined in the form of Fourier expansion as [24]

$$u=\sum_{n=1}(\overline{u}_n\cos n\theta+\overline{\overline{u}}_n\sin n\theta) \tag{3}$$

$$v=\sum_{n=1}(\overline{v}_n\sin n\theta-\overline{\overline{v}}_n\cos n\theta) \tag{4}$$

$$w=\sum_{n=1}(\overline{w}_n\cos n\theta+\overline{\overline{w}}_n\sin n\theta) \tag{5}$$

where n is the harmonic number. Single and double barred terms represent the symmetric and antisymmetric displacement amplitudes with respect to the plane $\theta=0$ [24].

If the applied load is symmetric about the plane $\theta=0$ and $n=1,2,3\ldots$, the terms of the single-barred series represent the