Finite Elements in Analysis and Design 86 (2014) 101–119

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

External coupling software based on macro- and micro-time scales for explicit/implicit multi-time-step co-computations in structural dynamics

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ARTICLE INFO

Article history:
Received 3 May 2013
Received in revised form
1 April 2014
Accepted 4 April 2014
Available online 4 May 2014

Keywords:
Non-overlapping coupling method
Heterogeneous time integrators
Heterogeneous time scales
External code coupling
Structural dynamics

ABSTRACT

External coupling software based on the coupling algorithm proposed by Prakash and Hjelmstad (PH method) is compared to the previous external coupling software based on the GC (Gravouil and Combsecure) method. The salient features of multi-time-step partitioning methods are presented: they involve non-overlapping partitions and follow a dual Schur approach by enforcing the velocity continuity at the interface with Lagrange multipliers. The main difference between the two methods lies in the time scale at which the interface problem is solved: the micro-time scale for the GC algorithm and macro-time scale for the PH algorithm. During the multi-time-step co-computations involving two finite element codes (explicit and implicit FE codes), the tasks carried out by the coupling software PH-CPL, based on a variant of the PH algorithm, are illustrated and compared to the coupling software GC-CPL based on the GC algorithm. The advantage of the new coupling PH-CPL software is highlighted in terms of parallel capabilities. In addition, the PH-CPL coupling software alleviates the dissipative drawback of the GC method at the interface between the subdomains. Academic cases are investigated to check the energy features and the accuracy order for the GC and PH algorithms. Finally, explicit/implicit multi-time-step co-computations with GC-CPL and PH-CPL software are conducted for two engineering applications under the assumption of linear elastic materials: a reinforced concrete frame structure under blast loading striking its front face and a flat composite stiffened panel subjected to localised loads applied to its central frame.

1. Introduction

The increasing complexity of numerical simulations involving multiphysics phenomena has boosted the development of partitioning approaches. In fluid–structure interactions, the partition is driven by both physical and computational considerations: phenomena expected in each medium are very different from the point of view of space and time scales, arguing for a natural partitioning of the problem. In structural dynamics such as a structure subjected to localised impact loads, a small part of the structure is expected to experience strong non-linear phenomena over a short duration along with high-frequency features, whereas the remaining structure would mainly be concerned with linear low-frequency vibrations over a much longer duration. Then the choice of the subdomain decomposition of the whole mesh is driven by computational consideration: adopting different time steps and time integrators (explicit or implicit) depending on the partitions of the structure is much more advantageous in terms of computation times than a fixed time step depending on the smallest finite element in the mesh when considering conditionally stable time integrators such as the explicit central difference scheme.

Partitioning approaches are also very appealing with a view to coupling dynamic parts of heterogeneous systems in the context of hybrid (experimental/numerical) real time testing which requires different explicit or implicit time integration schemes in different subdomains (i.e. substructures) depending on their complexity and characteristics. The purpose of such hybrid experimental/numerical testing such as continuous pseudo-dynamic testing (PsD) is to test a key region of interest while the remainder of the structure is numerically handled. Typically, fewer degrees of

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http://dx.doi.org/10.1016/j.finel.2014.04.005
0168-874X/

freedom are considered for the tested part but with a large number of small time steps, advocating the use of explicit time integration schemes or other non-iterative schemes such as the α-operator splitting method [1]. In contrast, the numerical part may involve a large number of degrees of freedom with a large time step, which is efficiently handled by an implicit time integration scheme. As a consequence, the hybrid numerical–experimental testing techniques require high-performance coupling methods from the field of computational mechanics to be able to trigger the physical and numerical parts to interact in real time.

A great deal of research on partitioning approaches has been published in the literature since the pioneering work of Hughes and Liu in 1978 [2,3], and Belytshko and Mullen in 1976 [4–6]. These time integration procedures followed a primal approach, that is, the solution of the kinematic quantities (primal field) is obtained by solving the interface problem. The procedures refer to mixed time integration methods when different time integrators are considered, and subcycling or multi-time-step methods when different subdomain time steps are used in the mesh. Domain decomposition is based on nodal partitioning or element partitioning [7–18].

The alternative approach to the primal subdomain decomposition methods is the dual approach, characterised by an interface problem whose unknowns are dual quantities, Lagrange multipliers, representing the interface forces between the subdomains. This approach has been popularised by the success of finite element tearing and interconnecting method (FETI) developed by Farhat and Roux [19], first for static problems and subsequently extended to dynamic problems [20]. The FETI method uses non-overlapping subdomains and provides a very useful framework for building new coupling algorithms such as the multi-time-step integration method proposed by Gravouil and Combescure, labelled the GC method [21]. The stability of the GC method is demonstrated for Newmark time integrators by studying the sign of the interface pseudo-energy using of the so-called energy method, as first proposed by Hughes [22] for demonstrating the stability of mixed time integration methods. The interface pseudo-energy remains equal to zero when the same time step is employed in the subdomains and becomes negative when different time steps are adopted. In terms of accuracy orders, the artificial energy dissipation results in the loss of one order of accuracy when second-order accurate Newmark time integrators with different time steps are coupled. Furthermore, the GC method proposed for Newmark schemes in linear dynamics has been extended to non-linear dynamics [23], explicit non-linear dynamics with non-matching meshes at the interface [24], and coupling subdomains described according to a modal approach [25]. Prakash and Hjelmstad [26] dealt with the energy dissipation issue at the interface by extending the GC method.

The main difference of the PH method lies in the time scale at which the interface problem is solved: contrary to the GC method whose interface problem is solved at the fine time scale, the interface problem is solved at the large time scale. It has been proven that the PH method is energy conserving in the sense that the interface pseudo-energy remains equal to zero. The partitioning method turns out to be stable and of second-order accuracy when Newmark second-order accurate time integrators are considered. In the context of first-order systems, a FETI-based domain decomposition technique has been developed by Nakshatrala et al. [27] enabling arbitrary numerical schemes to be coupled.

The development of hybrid experimental/numerical testing methods has fostered the tailoring of the previous subdomain methods, by investigating the suitability of primal [28] and dual [29] approaches for these particular heterogeneous subsystems. Indeed, the staggered nature of the GC method constitutes a severe drawback in the context of the PsD tests in which experimental and numerical tasks have to be conducted in a concurrent way. Consequently, a new interfield parallel integration procedure, called the PM method from the authors' names (Pegon and Magonnette), has been described by Bonelli et al. [30] by coupling Newmark time integrators. Recently, further work has been undertaken by Bursi et al. [31] to build a coupling method able to handle the generalised-α schemes proposed by Chung and Hulbert [32], well known for their optimal numerical dissipation properties by filtering out the high frequencies of the response without altering the low frequencies. Nonetheless, it has been pointed out that generalized-α schemes have to be adapted to the end time collocation format of the GC method along the lines of Arnold and Bruls [33].

In Section 2 of this paper, the salient features of the GC algorithm are reviewed as well as the discrete energy balance equation [22,34] in order to accurately assess the spurious energy dissipated at the interface. The next section is devoted to the PH method which enables the energy drawback at the interface to be alleviated, by solving the interface problem at the macro-time scale. It will be emphasised that some unbalanced free terms at the micro-time scale are required in the classical PH algorithm found in the literature [26,35]. An improved variant of the PH algorithm, noted as modified PH algorithm, is proposed by deriving a new form of these unbalanced free forces. In Section 4, on the basis of this modified PH algorithm, new coupling software, called PH-CPL, is set up and compared to the coupling software based on the GC algorithm (labelled GC-CPL). Finally, numerical examples are presented in the last section. The first example is the academic case of the split-oscillator to assess the accuracy orders of the GC and modified PH coupling schemes. The second example is a simple clamped-free 2D beam subjected to a lateral end load. The third example concerns a reinforced concrete frame structure subjected to blast loads at its front face, simulated using both coupling software programmes (GC-CPL and PH-CPL). The last example deals with a flat plane panel, with stiffeners and frames, subjected to localised impact loads. Multi-time-step explicit/implicit co-computations using both GC and modified PH coupling software programmes show the accuracy and efficiency of the partitioning approach in comparison to the full-explicit computation.

2. GC algorithm for subdomain coupling in dynamics

First, the key points of the GC progenitor multi-time-step subdomain algorithm proposed by Gravouil and Combescure [21,23] are given: the method is FETI-based, that is, it follows a dual Schur approach, involving the dual variable, the Lagrange multiplier, to enforce the kinematic continuity of the velocity across the interface at the micro-time scale.

Second, in order to accurately assess the dissipated energy at the interface due to the GC method, the discrete balance energy equation is written in the case of a domain split into two subdomains handled with different Newmark time integrators and time steps.

2.1. Multi-time-step GC method for two subdomains

The GC coupling method is based on a dual Schur complement formulation which consists in ensuring the velocity continuity across the interface between subdomains by means of Lagrange multipliers. The Lagrange multipliers represent the interface forces between the two subdomains Ω_A and Ω_B. The kinematic constraint, continuous over time, related to the velocity continuity across the interface is expressed as

$$L_A\,\dot{u}^A(t)+L_B\,\dot{u}^B(t)=0 \tag{1}$$