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A geometrically exact finite beam element formulation for thin film adhesion and debonding

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ABSTRACT

A nonlinear beam formulation is developed that is suitable to describe adhesion and debonding of thin films. The formulation is based on a shear-flexible, geometrically exact beam theory that allows for large beam deformations. The theory incorporates several aspects that have not been considered in previous theories before. Two different adhesion mechanisms are considered here: adhesion by body forces and adhesion by surface tractions. Corresponding examples are van der Waals adhesion and cohesive zone models. Both mechanisms induce a bending moment within the beam that can play an important role in adhesion and debonding of thin films. The new beam model is discretized within a nonlinear finite element formulation. It is shown that the new formulation leads to a symmetric stiffness matrix for both adhesion mechanisms. The new formulation is used to study the peeling behavior of a gecko spatula. It is shown that the beam model is capable of capturing the main features of spatula peeling accurately, while being much more efficient than 3D solid models.

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1. Introduction

The adhesion, debonding, and peeling behavior of thin strips and films plays an important role in many applications. Examples are paints and coatings, adhesive tapes, liquid films, and adhesive pads of insects and lizards like the gecko spatula pad. Since thin strips are slender, often also elongated, structures they are natural candidates for the consideration of beam theory. This is the basis of several analytical thin film peeling models that have been formulated starting with the seminal work of Kendall [1], see for example [2–5]. Analytical models are based on simplified assumptions regarding geometry and deformation. Thus they are not suitable to describe general problems characterized by the nonlinearities of large deformations and by complex geometries, as they are found in adhesive systems of insects and lizards. In these cases computational models are indispensable.

The objective of this paper, therefore, is to formulate a computational beam model for adhesion, debonding, and peeling. Here, we focus on a 2D formulation that is suitable to describe plane strain conditions of films, or to describe 2D behavior of beams. The considered formulation is based on the nonlinear, geometrically exact beam theory of Reissner [6], of which a computational counterpart is discussed in a book by Wriggers [7]. This formulation is generalized to beams with an initially curved axis and an arbitrary shaped cross section, which may vary along the beam. This is also accounted for in the presented contact formulation. The beam model is extended by two different adhesion formulations: adhesion by body forces and adhesion by surface tractions. The first is suitable to describe van der Waals adhesion, the second is suitable to include cohesive zone models. The formulation presented here is an extension of the van der Waals-based beam adhesion formulation of Sauer [8]. The new formulation accounts for both the shear deformation of the beam and a bending moment that is caused by the adhesion forces. Both these contributions have not been incorporated into a computational beam model before. It is seen that, combined, the two contributions lead to a symmetric finite element stiffness matrix. The symmetry is lost if one of the contributions is neglected. This symmetry reflects the fact that the model can be derived from a potential. The computational model presented here has been applied by Sauer [9] for studying the peeling behavior of shear-rigid beams with a rectangular cross section. The purpose of that study was to investigate the material and adhesion properties of thin peeling films, and to show that the bending stiffness can play a major role during peeling. It did, however, neither discuss the computational modeling nor the extension to shear-flexible beams with arbitrary, varying cross sections. This is the purpose of the present work.

The major advantage of the new formulation is the huge gain in efficiency it offers compared to adhesion models for 3D solids, like the model of Sauer and Wriggers [10]. To illustrate this, we compare the new beam formulation with the detailed 3D spatula model of Sauer and Holl [11], considering a vibration analysis and several peeling cases. It is seen that the beam model is capable of

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capturing the behavior of the spatula accurately. For the study, the characteristic beam properties (centroid, cross section, second moment of area, etc.) need to be determined from the detailed 3D geometry, which is discussed in detail. Since a van der Waals-based adhesion model leads to a purely normal (i.e. mode I) contact formulation, we also study gecko adhesion by considering a cohesive zone formulation that can describe tangential (mode II) debonding. In summary, it is shown that the new formulation is (1) more accurate than previous beam formulations, (2) consistent with continuum theory, and (3) a highly efficient alternative to 3D solid models.

The remaining sections of this paper are structured as follows. Section 2 gives an overview of the geometrically exact beam theory and shows how various adhesion formulations are adapted to the beam. Section 3 then presents the corresponding finite element formulation for adhesive beams. Numerical examples are discussed in Section 4. These consider the peeling behavior of a gecko spatula for various loading conditions and compare the beam results with detailed 3D computations based on solid elements. Section 5 concludes this paper.

2. Geometrically exact beam theory

This section presents the model equations governing the mechanical behavior of a thin adhesive strip. We discuss two different continuum adhesion models and their adaption to beam theory, focusing first on the internal work, $\delta\Pi_{\mathrm{int}}$, and then on the virtual contact work, $\delta\Pi_{\mathrm{c}}$.

2.1. Equilibrium equation

For adhesion, the (mechanical) weak form of the equilibrium equation is given by the following statement [10]: find an admissible deformation $\boldsymbol{\varphi}\in\mathcal{U}$ satisfying the principle of virtual work

$$\delta\Pi_{\mathrm{int}}+\delta\Pi_{\mathrm{c}}-\delta\Pi_{\mathrm{ext}}=0,\qquad \forall\,\delta\boldsymbol{\varphi}\in\mathcal{V}_{\varphi},\tag{1}$$

where $\delta\boldsymbol{\varphi}\in\mathcal{V}_{\varphi}$ denotes a kinematically admissible virtual deformation. The first term, $\delta\Pi_{\mathrm{int}}$, corresponds to the virtual work of the internal forces, see the following section. The second term, $\delta\Pi_{\mathrm{c}}$, which is discussed in Section 2.3, denotes the virtual work of contact and adhesion forces. The last term, $\delta\Pi_{\mathrm{ext}}$, denotes the virtual work of any external forces acting on the strip.

2.2. Kinematics and constitution

In the following, we outline the kinematics and constitution of the geometrically exact beam formulation of Reissner [6], see also [7]. This formulation accounts for the exact kinematics of large beam deformations and rotations. According to the assumptions of beam theory, only normal strains, due to axial forces and bending moments, and shear strains, due to shear forces, are considered. This means that the beam is supposed to be shear-flexible (like the Timoshenko beam). Further, the cross section of the beam is supposed to remain planar (but not necessarily normal to the beam axis) during deformation. Fig. 1 shows the nonlinear kinematics of the deforming beam. As shown, the beam axis is described by the coordinate S in the undeformed reference configuration, $\mathcal{B}_0$. The deformation of the beam is fully characterized by the three independent fields $u(S)$, $w(S)$, and $\psi(S)$, which denote the displacement of the beam axis and the rotation of the cross section (Fig. 1). The fields can be arranged in the vector

$$\boldsymbol{d}=\begin{bmatrix}u\\ w\\ \psi\end{bmatrix}.\tag{2}$$

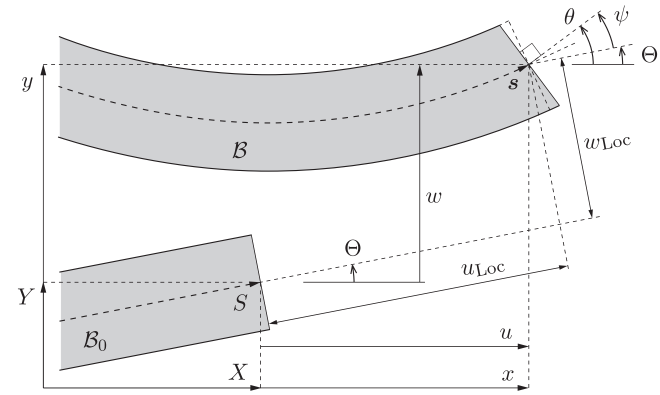

Fig. 1. Nonlinear kinematics of the geometrically exact beam.

The deformation of the beam is then characterized by

$$\boldsymbol{x}=\begin{bmatrix}x\\ y\\ \theta\end{bmatrix}=\begin{bmatrix}X\\ Y\\ \Theta\end{bmatrix}+\begin{bmatrix}u\\ w\\ \psi\end{bmatrix}=\boldsymbol{X}+\boldsymbol{d}.\tag{3}$$

This formulation corresponds to the global coordinate system shown in Fig. 1. The angle Θ corresponds to the initial inclination of the cross section, which may vary along S but must be perpendicular to the beam axis in the undeformed configuration. This follows from the theory of Reissner. Introducing the rotation

$$\mathbf{Q}(\theta)=\begin{bmatrix}\cos\,\theta & \sin\,\theta & 0\\ -\sin\,\theta & \cos\,\theta & 0\\ 0 & 0 & 1\end{bmatrix},\tag{4}$$

the displacement vector, $\boldsymbol{d}$, can be transformed to the local coordinate systems of the reference configuration, $\mathcal{B}_0$, and the current configuration, $\mathcal{B}$, as

$$\boldsymbol{d}_{\mathrm{Loc}}{:=}\mathbf{Q}(\Theta)\boldsymbol{d}\quad\text{and}\quad\boldsymbol{d}_{\mathrm{loc}}{:=}\mathbf{Q}(\theta)\boldsymbol{d}.\tag{5}$$

Note that $\mathbf{Q}(\theta)=\mathbf{Q}(\psi)\mathbf{Q}(\Theta)$. For the simplicity, we define a vector of partial derivatives, $\boldsymbol{d}'{:=}\partial\boldsymbol{d}/\partial S$. Introducing the transformations

$$\boldsymbol{d}'_{\mathrm{Loc}}{:=}\mathbf{Q}(\Theta)\boldsymbol{d}'\quad\text{and}\quad\boldsymbol{d}'_{\mathrm{loc}}{:=}\mathbf{Q}(\theta)\boldsymbol{d}',\tag{6}$$

the axial strain, ε, the shear strain, γ, and the flexure, κ, of the beam, arranged in the vector $\boldsymbol{\varepsilon}=[\varepsilon,\gamma,\kappa]^T$, can be either written as [7]

$$\boldsymbol{\varepsilon}=\mathbf{Q}(\psi)\boldsymbol{d}'_{\mathrm{Loc}}-\boldsymbol{\phi}\tag{7}$$

or equivalently as

$$\boldsymbol{\varepsilon}=\boldsymbol{d}'_{\mathrm{loc}}-\boldsymbol{\phi},\tag{8}$$

where $\boldsymbol{\phi}$ is defined as

$$\boldsymbol{\phi}=\begin{bmatrix}1-\cos\,\psi\\ \sin\,\psi\\ 0\end{bmatrix}.\tag{9}$$

Expression (8) provides a geometrically exact definition of the beam strains w.r.t. the local coordinate system. The strain definition according to Eq. (8) is the logical extension of the infinitesimal beam kinematics to large deformations and rotations.

The local axial force, N, shear force, V, and bending moment, M, of the beam follow from the chosen constitutive model. Here, we consider linear elastic material behavior, so the cross-sectional force $\boldsymbol{S}=[N,V,M]^T$ is given by

$$\boldsymbol{S}=\boldsymbol{D}\boldsymbol{\varepsilon}\tag{10}$$

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