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Static stability of laminated composite circular plates with holes using shear deformation theory

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Abstract

The behavior of buckling of laminated composite circular plates having circular holes and subjected to uniform radial load is investigated by using the finite element method. A finite element analysis program was developed to analyze static stability. Eight node isoparametric shell elements with 24 degrees of freedom are used during the investigation. The first-order shear deformation theory is used to consider the effects of the transverse shear deformation. The effects of hole sizes, location of the holes, thickness variations and boundary conditions on the buckling load of the composite plates are determined.

Keywords: Layered structures; The finite element analysis (FEA); Buckling (static stability); Plate theory

1. Introduction

Composite laminated plates are being increasingly used in modern engineering applications, due to their high strength-to-weight, stiffness-to-weight ratios and modulus [1–3]. The high specific strength and specific stiffness which are the bases of the superior structural performance of composite materials provide the composite materials many application choices. For moving parts, weight plays an important role in calculating the structural stability of the system. The fiber reinforced laminated composite plates with holes are used especially in weight-sensitive structures. Weight reduction is intended by hole opening and fuel, hydraulic and electrical lines can be placed through these holes. However, in such applications, buckling phenomenon may often be observed. Buckling is critical to structural components made of composite materials because the buckling of composite plates usually occurs at low applied loads compared to in-plane tensile loading and generates large deformation due to the small thickness/length ratio. Therefore, in design of these types of structures, static stability of laminated composite plates which are in general the main load carrying component of the system has to be analyzed.

The study of buckling of composite plates has been considered by some researchers. Sai Ram and Streedbar Babu [4] investigated the buckling of laminated composite shells subjected to transverse load. Kumar et al. [5] studied the tensile buckling, vibration and parametric instability behavior of doubly curved panels with central circular cutout subjected to uniaxial in-plane partially distributed tensile edge loadings using the finite element method. They used first-order shear deformation theory (FSDT) to model the curved panels, to consider the effects of the transverse shear deformation and rotary inertia. Akhras and Li [6] developed a spline finite strip method for static and free vibration analysis of composite plates using Reddy's high-order shear deformation theory. Shufrin and Eisenberger [7] presented analysis of the buckling loads for thick elastic rectangular plates with variable thickness and various combinations of boundary conditions. They applied both the first-order and high-order shear deformation plate theories to the plate's analysis. Huang and Shukla [8] obtained post-buckling behavior of cross-ply laminated composite plate containing randomly oriented short spatial fibers in each layer analytically, using

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fast converging double Chebyshev series. The mathematical formulation is based on FSDT and von-Karman nonlinearity. Xie et al. [9] proposed the buckling analysis of symmetrically laminated composite plates withth internal supports.

This paper intends to point out the effects of hole sizes, location of the holes on the buckling load of laminated composite circular plates.

2. Theoretical formulations

2.1. Strain–displacement relationships in cylindrical coordinates

In general, cylindrical coordinates (r, θ, z) (Fig. 1) are preferred over Cartesian coordinates (x, y, z) where a degree of axial symmetry exists both in geometry and loading. The laminate strains in cylindrical coordinates are of general form,

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial v_0}{\partial \theta} \\ \dfrac{1}{r}\dfrac{\partial w_0}{\partial \theta} + \phi_\theta \\ \dfrac{\partial w_0}{\partial r} + \phi_r \\ \dfrac{1}{r}\dfrac{\partial u_0}{\partial \theta} + \dfrac{\partial v_0}{\partial r} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \phi_r}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial \phi_\theta}{\partial \theta} \\ 0 \\ 0 \\ \dfrac{1}{r}\dfrac{\partial \phi_r}{\partial \theta} + \dfrac{\partial \phi_\theta}{\partial r} \end{Bmatrix}, \tag{1}$$

where u_0, v_0 and w_0 are midplane displacements in the r, θ and z directions and ϕ_r, ϕ_θ are the rotations of transverse normal about the r and θ axes, respectively.

2.2. Laminate constitutive equations

The laminate constitutive equations relate the force and moment resultants $(\boldsymbol{N}, \boldsymbol{M})$ to the midplane strains and curvatures $(\boldsymbol{\epsilon}^{(0)}, \boldsymbol{\epsilon}^{(1)})$. The force resultants are given by

$$\begin{Bmatrix} N_r \\ N_\theta \\ N_{r\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial v_0}{\partial \theta} \\ \dfrac{1}{r}\dfrac{\partial u_0}{\partial \theta} + \dfrac{\partial v_0}{\partial r} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \phi_r}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial \phi_\theta}{\partial \theta} \\ \dfrac{1}{r}\dfrac{\partial \phi_r}{\partial \theta} + \dfrac{\partial \phi_\theta}{\partial r} \end{Bmatrix} \tag{2a}$$

and the moment resultants are given

$$\begin{Bmatrix} M_r \\ M_\theta \\ M_{r\theta} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial v_0}{\partial \theta} \\ \dfrac{1}{r}\dfrac{\partial u_0}{\partial \theta} + \dfrac{\partial v_0}{\partial r} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \phi_r}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial \phi_\theta}{\partial \theta} \\ \dfrac{1}{r}\dfrac{\partial \phi_r}{\partial \theta} + \frac{\partial \phi_\theta}{\partial r} \end{Bmatrix}, \tag{2b}$$

where A_{ij} denote the extensional stiffnesses, D_{ij} the bending stiffnesses, and B_{ij} the bending–extensional coupling stiffnesses of a laminate and they can be calculated as

$$A_{ij} = \sum_{k=1}^{N} Q_{ij}^{(k)}(h_{k+1} - h_k), \quad B_{ij} = \frac{1}{2}\sum_{k=1}^{N} Q_{ij}^{(k)}(h_{k+1}^2 - h_k^2),$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} Q_{ij}^{(k)}(h_{k+1}^3 - h_k^3), \tag{3}$$

where $Q_{ij}^{(k)}$ are the material stiffness terms of the kth lamina, as referred to the laminate coordinates and N is the total number of layers in the laminate, and (h_k, h_{k+1}) are thickness coordinates of the bottom and top of kth layer (Fig. 2).

2.3. Equations of motion

The equations of motion of a solid body can be derived using either energy principles or vector mechanics. Energy principle has been used during this investigation. For the static case, the equilibrium equations of the FSDT are

$$\frac{\partial N_r}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial \theta} + q_{\mathrm{r}} = 0, \quad \frac{\partial N_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_\theta}{\partial \theta} = 0,$$

$$\frac{\partial Q_r}{\partial r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial \theta} + \frac{\partial}{\partial r}\left(N_r\frac{\partial w}{\partial r} + N_{r\theta}\frac{\partial w}{r\partial \theta}\right) + \frac{\partial}{r\partial \theta}\left(N_{r\theta}\frac{\partial w}{\partial r} + N_\theta\frac{\partial w}{r\partial \theta}\right) + q_{\mathrm{t}} = 0,$$

$$\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial \theta} - Q_r = 0, \quad \frac{\partial M_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial M_\theta}{\partial \theta} - Q_\theta = 0, \tag{4a}$$

where $(N_r, N_\theta, N_{r\theta})$ and $(M_r, M_\theta, M_{r\theta})$ are force and moment resultants defined in Eq. (2), q_{r} is the distributed radial force acting on disc, q_{t} is the distributed transverse force acting on disc and the quantities (Q_r, Q_θ) are called the transverse shear forces.