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Reynold's model viscosity on radiative MHD flow in a porous medium between two vertical wavy walls

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Abstract

The two dimensional heat transfer of a free convective–radiative MHD (magnetohydrodynamics) flows with variable viscosity and heat source of a viscous incompressible fluid in a porous medium between two vertical wavy walls was investigated. The fluid viscosity is assumed to vary as an exponential function of temperature. The flow is assumed to consist of a mean part and a perturbed part. The perturbed quantities were expressed in terms of complex exponential series of plane wave equation. The resultant differential equations governing the flow were non-dimensionalised and solved using Differential Transform Method (DTM). The numerical computations were presented in tabular and graphical forms for various fluid parameters. It shows that an increase in radiation, variable viscosity and permeability parameters cause a rise in velocity profile. Nusselt number increases with increase in heat source and decreases with increase in radiation parameter at both walls.

MSC: 76S05; 76W99

Keywords: Wavy walls; Porous medium; Reynold's model viscosity; Two-dimensional radiation; Differential Transform Method

1. Introduction

The study of viscous fluid flow over wavy wall(s) has received considerable attention of researchers due to its applications in nature and engineering such as in designing ventilated heating building, electronic components cooling and as well as in designing blood oxygenator and drying of several types of agricultural products (grain and food). In recent years, the interest of some researchers has been drawn to the study of fluid flow in the area of physiological processes through wavy channels. In particular, the peristaltic flow in the vasomotion of small blood vessels such as

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arterioles, venules, and capillaries, urine transport from kidney to bladder, spermatozoa transport in the ducts efferent of the male reproductive tract and in the cervical canal, transport of lymph in the lymphatic vessels, movement of ovum in the female fallopian tube and swallowing food through the oesophagus [1–7].

The theoretical and practical significance of the viscous fluid over wavy wall(s) were discussed by Lekoudis et al. [8], Shankar and Sinha [9], Lessen and Gangwani [10], Vajravelu and Sastri [11] and Das and Ahmed [12]. In the studies, the authors had given special attention to the wavines on the fluid flow and heat transfer characteristics.

The fluid flow through wavy channel is often used in certain engineering processes like glass manufacturing, crude oil refinement, paper production and in some geophysical studies under the influence of magnetic field [6,7]. The influence of magnetic field on corrugated channel was presented by Fasogbon [13] and it was reported that the magnetic field intensity slowed down the fluid motion. Tak and Kumar [14] investigated the heat transfer with radiation in the MHD free convection between a vertical wavy wall and a parallel flat wall. The authors presented that the thermal radiation speed up the velocity of the fluid. Fasogbon [15] considered the effects of mass transfer in irregular the channel and concluded that different chemical species enhance the velocity of the fluid. In the same vein, Abubakar [16] investigated natural convective flow and heat transfer in a viscous incompressible fluid with slip effect confined within spirally-enhanced channel and reported that the slip had a linear effect on the fluid motion. All these studies were narrowed down to one vertical wavy wall with a parallel flat wall.

The study of MHD free convection flow between two vertical wavy walls was studied by Tak and Kumar [17] and Kumar [18]. The authors noticed for non porous medium that viscous dissipation and thermal radiation had a linear accelerating effect on the fluid flow while the magnetic field intensity and heat source had a retarding effect on the fluid motion.

The study of fluid flow in a porous medium is significant because of its applications in soil mechanics, water purification, underground water hydrology, chemical engineering, petroleum engineering, ceramic engineering, metallurgical engineering, agricultural engineering and water irrigation process. Hence, Adesanya and Makinde [19], Makanda et al. [20] and Das et al. [21] had studied the fluid flow in a porous medium between two parallel flat walls.

However, the study of the fluid flow in a porous between two wavy walls has received relatively less attention. Among the studies, Teneja and Jain [22] investigated the MHD free convection flow in the presence of a temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall in slip flow regime with constant heat flux at the flat wall through a porous medium between vertical wavy walls. Dada and Disu [23] studied the heat transfer with radiation and temperature dependent heat source in a porous medium between two vertical wavy walls. It was reported that the velocity of the fluid increases with the increase in the permeability of the porous medium.

In all the above-mentioned studies, viscosity of the fluid was assumed to be constant. However, It is a known fact that viscosity property varies significantly with temperature [24]. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid. Hence, assuming a uniform viscosity throughout the fluid flow regime may not represent the physical reality of the flow.

As a result, the present study investigates the Reynold's model (temperature dependent) viscosity on a radiative MHD free convection flow in a porous medium between two vertical wavy walls. To achieve the aim of this study, perturbation technique coupled with Differential Transform Method was used to obtain the set of solutions of non-linear differential equations.

2. Formulation of the problem

Consider a steady laminar natural convective hydromagnetic flow in a porous medium between a long vertical wavy channel. The X-axis is vertically upwards while Y-axis is perpendicular to it. The wavy walls are represented by $Y=\varepsilon^*\cos(\Lambda X)$ and $Y=a+\varepsilon^*\cos(\Lambda X)$ respectively, where $\varepsilon^*\ll 1$, and Λ is the amplitude of the wavy walls. The equations governing the steady flow and heat transfer with temperature dependent heat source are as follows:

$$\rho\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{\partial}{\partial X}\left(\mu(T)\frac{\partial U}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\mu(T)\frac{\partial U}{\partial Y}\right)+g\beta(T-T_c)-H_0^2U-\frac{\mu(T)}{K^*}U, \quad (1)$$

$$\rho\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\frac{\partial}{\partial X}\left(\mu(T)\frac{\partial V}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\mu(T)\frac{\partial V}{\partial Y}\right)-\frac{\mu(T)}{K^*}V, \quad (2)$$

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