



Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability

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Abstract

In the present study, by implementing the direct algebraic method, we present the traveling wave solutions for some different kinds of the Korteweg–de Vries (KdV) equations. The exact solutions of the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained. Solutions for the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained precisely and efficiency of the method can be demonstrated. The stability of these solutions and the movement role of the waves by making the graphs of the exact solutions are analyzed. All solutions are exact and stable, and have applications in physics.

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Keywords: Traveling wave solutions; Extended direct algebraic method; Kawahara; Fifth order KdV; Generalized KdV equations

1. Introduction

Nonlinear wave phenomena exist in many fields, such as fluid mechanics, plasma physics, biology, hydrodynamics, solid state physics and optical fibers [1–4]. In order to better understand these nonlinear phenomena, it is important to seek their exact solutions. They can help to analyze the stability of these solutions and the movement role of the wave by making the graphs of the exact solutions [5–9]. The KdV equation plays an important role in describing motions of long waves in shallow water under gravity, one-dimensional nonlinear lattice, fluid mechanics, quantum mechanics, plasma physics and nonlinear optics [9–12].

There are many classical methods proposed to solve the KdV equations, including direct integration, direct algebraic approach, Lyapunov approach, Hirota's dependent variable transformation, the inverse scattering transform, and the

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Bäcklund transformation. Analytical exact solutions of a nonlinear evolution equation in mathematical physics; namely the time-fractional fifth-order Sawada–Kotera equation by the tanh–sech method via fractional complex transform were constructed [5]. Wazwaz considered the abundant solitons solutions, compactons and solitary patterns solutions, some new solitons and periodic solutions of the fifth-order KdV equation [13,14]. Sierra and Salas used a generalization of the well-known tanh–coth method to obtain new periodic and soliton solutions for several forms of the fifth-order KdV equation [15]. Using the Lie group analysis method, the invariance properties of the time fractional generalized fifth-order KdV equation were studied [16].

An extended simplest equation method was proposed to seek exact travelling wave solutions of nonlinear evolution equations. As applications, many new exact travelling wave solutions for several forms of the fifth-order KdV equation were obtained. The forms include the Lax, Sawada–Kotera, Sawada–Kotera–Parker–Dye, Caudrey–Dodd–Gibbon, Kaup–Kupershmidt, Kaup–Kupershmidt–Parker–Dye, and the Ito forms [17]. The Exp-function method was applied to obtain new generalized solitary solutions and periodic solutions of the fifth-order KdV equation [18–22]. The 1-soliton solution of three variants of the generalized KdV equation with generalized evolution were obtained [23,24]. Soliton solutions to KdV equation with spatio-temporal dispersion were given [25]. Additional conservation laws for Rosenau–KdV–RLW equation with power law nonlinearity by Lie symmetry was obtained [26]. Exact and explicit solutions to some nonlinear evolution equations by utilizing the (G'/G) -expansion and extended direct algebraic methods were given [27–31].

This paper is organized as follows: an introduction is given in Section 1. In Section 2, an analysis of the extended direct algebraic method is formulated. In Sections 3 and 4, the exact solutions of the Kawahara, the fifth order KdV and generalized fifth order KdV equations are obtained. Finally the paper end with a conclusion is given in Section 5.

2. An analysis of the extended direct algebraic method

The following is a given nonlinear partial differential equations (KdV equations) with two variables x and t [32] as

$$F(u, u_t, u_x, u_{xx}, u_{xxx}, u_{xxxx}) = 0, \quad (1)$$

where F is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformations.

Step 1: Assume that Eq. (1) has the following formal solution as:

$$u(x, t) = u(\xi) = \sum_{i=0}^m a_i \varphi^i(\xi), \quad (2)$$

where

$$\varphi' = \sqrt{\alpha\varphi^2 + \beta\varphi^4} \quad \text{and} \quad \xi = kx + \omega t, \quad (3)$$

where α, β , are arbitrary constants and k and ω are the wave length and frequency.

Step 2: Balancing the highest order derivative term and the highest order nonlinear term of Eq. (1), and the coefficients of series $\alpha, \beta, a_0, a_1, a_m, k, \omega$ are parameters can be determined.

Step 3: Substituting from Eqs. (2) and (3) into Eq. (1) and collecting coefficients of $\varphi^i \varphi^{(i)}$, then setting coefficients equal zero, we will obtain a set of algebraic equations. By solving the system, the parameters $\alpha, \beta, a_0, a_1, a_m, k, \omega$ can be determined.

Step 4: By substituting the parameters $\alpha, \beta, a_0, a_1, a_m, k, \omega$ and $\varphi(\xi)$ obtained in step 3 into Eq. (2), the solutions of Eq. (1) can be derived.

3. Stability analysis

Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. By using the form of a Hamiltonian system for which the momentum is given as

$$M = \frac{1}{2} \int_{-\infty}^{\infty} u^2 d\xi, \quad (4)$$

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