Title: A Shifted Legendre method with residual error estimation for delay linear Fredholm integro-differential equations

Author: Şuayip Y*ü*zbaşi

PII: S1658-3655(16)30001-2
DOI: http://dx.doi.org/doi:10.1016/j.jtusci.2016.04.001
Reference: JTUSCI 290

To appear in:

Received date: 10-1-2016
Accepted date: 1-4-2016

Please cite this article as: Ş. YÜZBAŞI, A Shifted Legendre method with residual error estimation for delay linear Fredholm integro-differential equations, *Journal of Taibah University for Science* (2016), http://dx.doi.org/10.1016/j.jtusci.2016.04.001

A Shifted Legendre method with residual error estimation for delay linear Fredholm integro-differential equations

Şuayip YÜZBAŞI*

Department of Mathematics, Faculty of Science, Akdeniz University, 07058, Antalya, Turkey
E-mail: syuzbasi@akdeniz.edu.tr

Abstract: In this paper, we suggest a matrix method for the approximate solutions of the delay linear Fredholm integro-differential equations with constant coefficients by using the shifted Legendre polynomials. The problem is considered with the mixed conditions. By using the required matrix operations, the delay linear Fredholm integro-differential equation is transformed into a matrix equation. Also, error analysis for method is presented by using the residual function. The illustrative examples are given to demonstrate the efficiency of the method. The obtained results are compared by the known results.

Mathematics Subject Classification: 45B05; 45L05; 65L03; 34B05; 65L80

Keywords: *Delay Fredholm integro-differential equations, Numerical solution, Matrix method, Shifted Legendre polynomials.*

1. Introduction

Delay differential equations and delay integro differential equations have major importance in many applied areas including engineering, mechanics, physics, chemistry, astronomy, biology, economics, potential theory, electrostatics, etc. Therefore, in this paper, we present a new matrix method for solutions of the delay linear Fredholm integro-differential equations with constant coefficients in the form

$$\sum_{k=0}^{m} F_k y^{(k)}(x+\tau_k) = g(x) + \int_0^1 \sum_{s=0}^{m} K_s(x,t) y^{(s)}(t+\gamma_s)dt, \quad 0 \le x,t \le 1 \tag{1}$$

under the mixed conditions

$$\sum_{k=0}^{m-1}\left(a_{jk} y^{(k)}(0) + b_{jk} y^{(k)}(1)\right) = \mu_j,\ j = 0,1,...,m-1 \tag{2}$$

where F_k, a_{jk}, b_{jk}, τ_k, γ_s and μ_j are real constants, $y^{(0)}(x) = y(x)$ is an unknown function, $g(x)$ and $K_s(x,t)$ are the functions defined on interval $0 \le x,t \le 1$ and also, $g(x)$ and $K_s(x,t)$ can be represented by Maclaurin series.

Our purpose in this study is to find the approximate solutions of Eq.(1) under conditions (2) in the form

$$y_N(x) = \sum_{n=0}^{N} a_n L_n(x). \tag{3}$$

Here, a_n $(n = 0,1,2,...,N)$ are the unknown Legendre coefficients and $L_n(x)$, $(n = 0,1,2,...)$ denote the shifted Legendre polynomials, which are defined by (Rodriguez formula)

$$L_n(x) = \frac{1}{n!}\frac{d^n}{dx^n}(x^2-x)^n,\ n \in \mathbb{N};\ 0 \le x \le 1.$$

In the recent years, many authors have studied on the numerical methods for the approximate solutions of integro-diferantial equations [1-14].

* Corresponding author. Tel.: + 90 242 310 23 83 fax: + 90 242 227 89 11.
E-mail address: syuzbasi@akdeniz.edu.tr (Ş. YÜZBAŞI).