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Qualitative properties of certain non-linear differential systems of second order

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Abstract

In this paper, we study the boundedness and square integrability of solutions in certain non-linear systems of differential equations of second order. We establish two new theorems, which include suitable sufficient conditions guaranteeing the boundedness and square integrability of solutions to the considered systems. The presented proofs simplify previous works since the Gronwall inequality is avoided which is the usual case. The technique of proof involves the integral test, and two examples are included to illustrate the results.

Keywords: Boundedness; Square integrability; Integral test; Non-linear; Differential system; Second order

1. Introduction

The study of qualitative behaviors of solutions, asymptotic behavior, stability, instability, boundedness, convergence, square integrability, etc., to differential equations of second order seems to be an important problem of the qualitative differential equations theory and has both theoretical and practical values in the literature. Numerous works were done on the subject (for example, see, the books or the papers of Ahmad and Rama Mohana Rao [1], El-Sheikh et al. [2], Gallot et al. [3], Grigoryan [4], Gu and Yu [5], Korkmaz and Tunc [6], Kroopnick [7], Pettini and Valdettaro [8], Kulcsar [11], Ogundare et al. [12], Sanchez [13], Shi [14], Tunc [15–18], Tunc and Tunc [19,20], Zhao [21] and the cited papers or books therein). However, we would not like to give here the details of the works and applications done regarding the mentioned qualitative properties.

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More recently, Napoles Valdes [9] considered the following second order scalar non-linear differential equation of the form

$$(p(t)x')' + f(t, x, x')x' + a(t)g(x) = q(t, x, x'). \tag{1}$$

Napoles Valdes [10] proved two new results concerning the boundedness and square integrability of the solutions of Eq. (1), under suitable assumptions. By this work, the author extended and improved the current literature through the sound mathematical analysis.

In [10], Napoles Valdes proved the following two theorems, respectively.

Theorem A ((Napoles Valdes [10,Theorem 1])). Assume that the following conditions hold:

(i) p and a are continuous functions on $I=[0, \infty)$ such that $0<p_0\le p(t)<+\infty$ and $a_0\le a(t)\le a_1<+\infty$.
(ii) f is a continuous function on $I\times\Re^2$ satisfying $0<f_0\le f(t, x, x')$.
(iii) g is also a continuous function for all x such that $xg(x)>0$ for $x\neq 0$ and $\int_0^{+\infty} g(x)dx = \infty$.
(iv) $|q(t, x, x')| \le e(t)$, where $e(t)$ is a non-negative and continuous function of t and satisfying $\int_0^{\infty} e(t)dt \le M < +\infty$, M is a constant. Then any solution $x(t)$ of Eq. (1), as well as its derivative, is bounded as $t\to\infty$ and $\int_0^{\infty} x'^2(t)dt < +\infty$.

Theorem B ((Napoles Valdes [10,Theorem 2])). Under hypotheses of Theorem A, we suppose that $xg(x)>g_0x^2$ for some positive constant $g_0>0$, and $0<p<p(t)<P<+\infty$, then all solutions of Eq. (1) are L^2 – solutions.

Napoles Valdes [10] proved both of Theorem A and Theorem B by the integral test.

In this paper, we take into consideration the following non-linear vector differential equations of the second order:

$$(q(t)X')' + H(t, X, X')X' + a(t)X = Q(t, X, X') \tag{2}$$

and

$$(q(t)X')' + \Phi(t, X, X') + a(t)G(X) = Q(t, X, X'), \tag{3}$$

respectively, where $X\in\Re^n$, $t\in\Re^+$, $\Re^+=[0, \infty)$; $H(.)$ is an $n\times n$ – symmetric and real valued continuous matrix, $q(.):\Re^+\to\Re$, with $q'(t)$ exits and is continuous, $a(.):\Re^+\to\Re$, $\Phi(.):\Re^+\times\Re^n\times\Re^n\to\Re^n$, and $Q(.):\Re^+\times\Re^n\times\Re^n\to\Re^n$ are continuous functions.

The purpose of this paper is to improve and extend the obtained results of [10] for Eq. (1) to Eqs. (2) and (3). In addition, we give the proofs related to the boundedness and square integarble solutions and their derivatives, which are less complex and quite general than those in the literature, by the integral test. In addition, two examples are presented to illustrate and verify the applicability of the obtained results. This paper has a new contribution to the topic in the literature. This fact shows the novelty and originality of this paper. The results to be established here may be useful for researchers working on the qualitative theory of solutions.

2. Boundedness

The following lemma plays a key role in proving our main results.

Lemma ((Horn and Johnson [9])). Let A be a real symmetric $n\times n$ – matrix. Then, for any $X\in\Re^n$,

$$a_1||X||^2 \ge \langle AX, X\rangle \ge a_0||X||^2,$$

where a_0 and a_1 are the least and greatest eigenvalues, respectively, of A.

The first main problem of this paper is the following theorem.

Theorem 1. Given Eq. (2). We assume that there exist positive constants a_0, a_1, h_0 and q_0 such that the following assumptions hold:

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