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A new cylindrical element formulation and its application to structural analysis of laminated hollow cylinders

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Abstract

Design and application of superelements in efficient prediction of the structural behavior in a short time has been one of the research interests in the last decade. Superelements are those elements which are capable of producing the same accurate result instead of several combined simple elements. In this paper a new 16-node cylindrical superelement is presented. Static and modal analyses of laminated hollow cylinders subjected to various kinds of loadings and boundary conditions are performed using this element. Some numerical examples are given to illustrate and validate the developed method. The accuracy of the results is evident from comparison of the findings with exact solution method and conventional finite element. It is seen that this element can predict the structural behavior of laminated cylinders in complex loading and boundary conditions in an efficient manner.

Keywords: FEM; Superelement; Hollow cylinder; Cylindrical element; Modal analysis

1. Introduction

The finite element method (FEM) provides a mathematically simplified procedure to simulate and analyze complex structures. However, the FEM becomes time consuming as the number of elements meshing the entire structure increases. Decreasing the number of elements and degrees of freedom (DOF) without altering the accuracy of the findings will result in reduction of the required storage capacity and run time consumption. Several methods to reduce the number of DOFs and increase the computational efficacy in both static and dynamic analyses of solid structures have been presented in the past. The spectral finite element method (SFEM) [1] and the transfer matrix method [2] are examples of these efforts. Development and application of superelements in structural analysis of various mechanical systems have been widely extended in the last decade. Ju and Choo [3] developed a superelement for modelling a cable passing through multiple pulleys. Jiang and Olson [4] extended a superelement to the nonlinear static and dynamic analysis of orthogonally stiffened cylindrical shells. Application of geometrical superelements in large deformation of elasto-plastic shells is presented by Lukasiewicz [5]. Using superelements has also been usual in structural analysis of buildings. Kim and Lee [6] used a rectangular superelement to analyze shear walls with openings in a building. Many researchers have used superelements in static and dynamic analyses of stiffened shells and plates. Koko and Olson [7] used a rectangular superelement, which is a macro-element having analytical as well as the usual finite element shape functions for vibration analysis of isotropic stiffened plates. Koko and Olson [8,9], Jiang and Olson [10,11], Vaziri et al. [12] have used this element for the analysis of plates and shells. Ahmadian and Zangeneh [13,14] have implemented this element into the dynamic analysis of laminated composite plates. Laminated cylinders are basic members in many structures and mechanical systems. They are used as the main part of motors and rotary systems. Hence accurate prediction of their behavior is of considerable importance. Axisymmetric element formulation is well-known as presented in the work by Hughes [15]. In this kind of formulation the three-dimensional problem reduces to a two-dimensional one, but demands that both the loads and boundary conditions should be symmetrical.

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Some specific elements in ANSYS like the PIPE element are also useful in modelling cylindrical structures but not able to model laminated cylinders. The PIPE element is not also capable of predicting the accurate stress contour in the cylinder. Using the new cylindrical superelement, neither loads nor the boundary conditions need to be axisymmetric, so a wide range of problems may be analyzed. In this paper the new cylindrical superelement is introduced. The shape functions are presented and the procedure of extracting the element stiffness and mass matrices is completely illustrated. Using the element matrices, several loading and boundary conditions are investigated for single layer and multilayer cylinders to perform static and modal analysis, and the resulted deformations, stresses and mode shapes are compared against conventional finite element.

2. Element definition

Consider a cylindrical element with length, $2L$, inner radius r_1 and outer radius r_2, which is depicted in Fig. 1. As it is shown, there are 16 nodes in each element distributed equally on both sides.

2.1. Intrinsic coordinates

Since the element is three dimensional, three independent coordinates are required to completely describe the position vector in the element. In the global coordinate system the triad, cylindrical coordinate (r, α, z) is used to determine the position vector in the element where r, α and z are radial, tangential and axial coordinates, respectively, as shown in Fig. 1. But the element formulation is completely done based on the intrinsic coordinate system which is a local system based on each individual element. The intrinsic coordinate system is stated as (ξ, η, γ), and defined as follows:

$$\xi = \frac{z}{L}, \quad \eta = \frac{2r - b}{a}, \quad \gamma = \frac{\alpha}{\pi} - 1, \tag{1}$$

where

$$a = r_2 - r_1, \quad b = r_2 + r_1 \tag{2}$$

and $2L$ is the element length.

Considering the appropriate limits for global coordinates:

$$-L \leqslant z \leqslant L, \quad r_1 \leqslant r \leqslant r_2, \quad 0 \leqslant \alpha \leqslant 2\pi \tag{3}$$

implies

$$-1 \leqslant \xi, \eta, \gamma \leqslant 1. \tag{4}$$

2.2. Shape functions

The following conditions are necessary for the shape functions of the cylindrical superelement.

The shape functions should be continuous and differentiable.

There should be one and only one shape function corresponding to each node that is 1 at that node and vanishes at other nodes.

According to the cylindrical shape of the element, all the shape functions, N_i should be periodic with respect to the circumferential coordinate α, with a period of 2π which implies a period of 2 with respect to local coordinate γ according to Eq. (1).

Satisfying the above-mentioned conditions the following shape functions are resulted which are used to approximate the field variable in the element:

$$\begin{aligned}
N_1(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma - \cos\pi\gamma)(1+\xi)(1+\eta),\\
N_2(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma - \cos\pi\gamma)(1-\xi)(1+\eta),\\
N_3(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma - \sin\pi\gamma)(1+\xi)(1+\eta),\\
N_4(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma - \sin\pi\gamma)(1-\xi)(1+\eta),\\
N_5(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma + \cos\pi\gamma)(1+\xi)(1+\eta),\\
N_6(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma + \cos\pi\gamma)(1-\xi)(1+\eta),\\
N_7(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma + \sin\pi\gamma)(1+\xi)(1+\eta),\\
N_8(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma + \sin\pi\gamma)(1-\xi)(1+\eta),\\
N_9(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma - \cos\pi\gamma)(1+\xi)(1-\eta),\\
N_{10}(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma - \cos\pi\gamma)(1-\xi)(1-\eta),\\
N_{11}(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma - \sin\pi\gamma)(1+\xi)(1-\eta),\\
N_{12}(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma - \sin\pi\gamma)(1-\xi)(1-\eta),\\
N_{13}(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma + \cos\pi\gamma)(1+\xi)(1-\eta),\\
N_{14}(\xi,\eta,\gamma) &= \frac{1}{8}(\cos^2\pi\gamma + \cos\pi\gamma)(1-\xi)(1-\eta),\\
N_{15}(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma + \sin\pi\gamma)(1+\xi)(1-\eta),\\
N_{16}(\xi,\eta,\gamma) &= \frac{1}{8}(\sin^2\pi\gamma + \sin\pi\gamma)(1-\xi)(1-\eta),
\end{aligned} \tag{5}$$

where ξ, η and γ are the intrinsic coordinates defined in Eq. (1).

3. Stress–strain relations

The displacement vector in an interior point of the element is

$$\mathbf{u} = [u_r \quad u_\alpha \quad u_z]^{\mathrm{T}} \tag{6}$$

which is obtained using the shape functions and the nodal displacement vector, $\mathbf{q}$, according to

$$\mathbf{u} = \mathbf{N}\mathbf{q}, \tag{7}$$

where

$$\mathbf{q} = [u_{1r} \quad u_{1\alpha} \quad u_{1z} \quad \ldots \quad u_{16r} \quad u_{16\alpha} \quad u_{16z}]^{\mathrm{T}} \tag{8}$$